Hashing

1. Hash Functions
   - mapping data to tables
   - hashing integers and strings

2. Open Addressing
   - a class Hash_Table
   - inserting and locating strings

3. Chaining
   - a class Hash_Table
   - inserting and locating strings

MCS 360 Lecture 28
Introduction to Data Structures
Jan Verschelde, 1 November 2017
1 Hash Functions
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mapping data to tables

We covered STL sets and maps for a frequency table. The STL implementation uses a balanced search tree.

An alternative to using trees is hashing. As running application, consider frequency tables.

We map counted objects first to a positive integer: the key, and then we take $key \mod n$, for tables of size $n$.

Using a hash function $h$ to store objects in table of size $n$:

$$object \xrightarrow{h(\cdot)} key \xrightarrow{\% n} index$$

The $index$ is used in an array of $n$ elements.
Hash Functions

The mapping of an object (e.g.: integer or string) to a key is done by a hash function $h$.

Desirable properties:

1. $h$ is fast and easy to evaluate;
2. few collisions: $x \neq y \Rightarrow h(x) \neq h(y)$.

Some similarity with uniform random number generator, although distribution of data is often not uniform, we want distribution of the keys to be uniform.
unsigned int

The type used to index arrays is `size_t`, which is 8 bytes long on 64-bit configurations, or otherwise 4 bytes long.

Including `<cmath>` allows

```cpp
int n = 8*sizeof(unsigned int);
unsigned int m = pow(2.0, n-1);
cout << " m = " << m << endl;
m = 2*m;
cout << "2*m = " << m << endl;
m = m - 1;
cout << "2*m - 1 = " << m << endl;
```

shows the modular arithmetic (modulo $2^{32}$)

```
m = 2147483648
2*m = 0
2*m - 1 = 4294967295
```
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Hashing Integers

Take a prime \( p \), \( h(i) = (p \times i) \mod 2^{32} \).
Start with \( i = 1 \), find \( n \) so \( p^n \mod 2^{32} = 1 \)?

```
unsigned int x = 1;
unsigned int i;
while(true)
{
    x = prime*x; i++;  
    if(x == 1)
    {
        cout << "cycle detected at i = "
             << i << endl;
        break;
    }
}

cycle detected at i = 536870912
```
hashing strings

For a string $s = s_0 s_1 s_2 \cdots s_{n-1}$, compute

$$k = s_0 31^{n-1} + s_1 31^{n-2} + s_2 31^{n-3} + \cdots + s_{n-1}$$

$$= (\cdots ((s_0 31 + s_1) 31 + s_2) 31 + \cdots ) 31 + s_{n-1}.$$  

Characters are 8 bits long; 31 is prime.

```cpp
size_t hash ( string s )
{
    size_t r = 0;

    for(size_t i=0; i<s.length(); i++)
        r = r*31 + s[i];

    return r;
}
```
open addressing and chaining

The two ways to organize hash tables differ in the ways of resolving collisions.

For some index, we consider

1. open addressing: if index occupied take next one; Locating an element is called probing.

2. chaining: keep list of elements at position index. The list is called a bucket, we speak of bucket hashing.
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We define a class **Hash_Table** in the namespace `mcs360_open_hash_table`.

The hash table stores a frequency table:
- type of the key is a string,
- value type is an integer.

```cpp
typedef std::pair<std::string, int> entry;
```

We create hash table with number of elements $n$, the capacity of the table.
mcs360_open_hash_table.h

#ifndef MCS360_OPEN_HASH_TABLE_H
#define MCS360_OPEN_HASH_TABLE_H
#include <utility>
#include <string>
#include <vector>
namespace mcs360_open_hash_table
{
    class Hash_Table
    {
    public:
        typedef std::pair<std::string, int> entry;
        Hash_Table(size_t n);
        bool insert(const entry& e);
        // returns true if e is inserted well
        int value(std::string s);
        // returns -1 if s is not found
    }
}
private members

private:

    size_t hash ( std::string s );
    // returns an index in the table
    // applying a hash function to the string s

    std::vector<entry*> table;
    // data member

Our hash table will have a fixed capacity, determined at instantiation.

Enlarging the hash table requires rehashing.
Hash_Table T(n);

while(true){ // inserting
    string w; cin >> w;
    Hash_Table::entry e;
    e.first = w;
    cin >> e.second;
    if(T.insert(e)) // rest omitted
}

while(true){ // searching
    string w; cin >> w;
    int v = T.value(w);
    if(v == -1) // rest omitted
}
constructor and hash function

Hash_Table::Hash_Table( size_t n )
{
    for(int i=0; i<n; i++)
        table.push_back(NULL);
}

type Hash_Table::hash ( std::string s )
{
    size_t r = 0;

    for(size_t i=0; i<s.length(); i++)
        r = r*31 + s[i];

    return (r % table.size());
}
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The `insert()` method in the context of Hashing is defined as follows:

```cpp
bool Hash_Table::insert( const entry& e ) {
    size_t i = hash(e.first);
    if(table[i] == NULL) {
        table[i] = new entry(e);
        return true;
    } else if(table[i]->first == e.first) {
        table[i]->second = e.second;
        return true;
    } else {
        for(int j=1; j<table.size(); j++) {
            size_t k = (i+j) % table.size();
            if(table[k] == NULL) {
                table[k] = new entry(e);
                return true;
            }
        }
    }
    return false;
}
```
int Hash_Table::value( std::string s ) {
    size_t i = hash(s);
    if(table[i] == NULL)
        return -1;
    else if(table[i]->first == s)
        return table[i]->second;
    else {
        for(int j=1; j<table.size(); j++) {
            size_t k = (i+j) % table.size();
            if(table[k] == NULL)
                return -1;
            else if(table[k]->first == s)
                return table[k]->second;
        }
        return -1;
    }
    return -1;
}
Advantage: no other data structure needed to deal with collisions.

Disadvantages:

- clusters may overlap;
  alternative to linear probing: quadratic probing
  in quadratic probing, increments form quadratic series:

\[ 1^2 + 2^2 + 3^2 + \ldots + i^2 \]

- deleted items must be marked and cannot be deallocated because of open addressing.
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In the namespace `mcs360_chain_hash_table` we define a `Hash_Table`.

The hash table stores a frequency table:
- type of the key is a string,
- value type is an integer.

```cpp
typedef std::pair<std::string, int> entry;
```

We create a hash table with number of elements $n$, the capacity of the table.
#ifndef MCS360_CHAIN_HASH_TABLE_H
#define MCS360_CHAIN_HASH_TABLE_H
#include <utility>
#include <string>
#include <vector>
#include <list>
namespace mcs360_chain_hash_table
{
    class Hash_Table
    {
    public:
        typedef std::pair<std::string, int> entry;
        Hash_Table( size_t n );
        bool insert( const entry& e );
        // returns true if e is inserted well
        int value( std::string s );
        // returns -1 if s is not found
    };
}
#endif // MCS360_CHAIN_HASH_TABLE_H
private members

The public part of this *Hash_Table* is the same as with open addressing

⇒ use the same interactive test

private:

```cpp
size_t hash ( std::string s );
// returns an index in the table
// applying a hash function to the string s

std::vector< std::list<entry> > table;
// data member
```
constructor and hash function

Hash_Table::Hash_Table( size_t n )
{
    for(int i=0; i<n; i++)
    {
        std::list<entry> L;
        table.push_back(L);
    }
}

size_t Hash_Table::hash ( std::string s )
{
    size_t r = 0;

    for(size_t i=0; i<s.length(); i++)
    {
        r = r*31 + s[i];
    }

    return ( r % table.size() );
}
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the \texttt{insert() method}

\begin{verbatim}
bool Hash_Table::insert( const entry& e )
{
    size_t i = hash(e.first);

    table[i].push_back(e);

    return true;
}
\end{verbatim}
search a value

```cpp
int Hash_Table::value( std::string s )
{
    size_t i = hash(s);

    if(table[i].size() == 0)
        return -1;
    else
    {
        std::list<entry> L = table[i];

        for(std::list<entry>::const_iterator k = L.begin(); k != L.end(); k++)
            if(k->first == s) return k->second;

        return -1;
    }
}
```
Resolving collisions with chaining:
- collisions affect only collided keys,
- flexible insert and effective delete,
- at expensive of list data structure.

The STL list takes care of bucket management.

We covered very basic implementations, left to do:
1. write delete methods
2. templates for key and value type
3. iterators on the tables
Summary + Exercises

Covered more of chapter 9, elements of §9.3, 9.4, and 9.5.

Exercises:

1. Download a text from www.gutenberg.org and write code to read the words from file. Use the STL map to make a frequency table of the keys applying on the strings the function hash of this lecture. Study the number of collisions for various sizes of the table.

2. To hash strings, would it help to apply the hash function \( h(i) = (p \times i) \mod 2^{32} \) on the key returned by the hash function on strings? Justify your answer, eventually referring to your answer to the previous exercise.
more exercises

3 Run the code on a small table with 3 entries to simulate collisions. Add printing statements and sketch the evolution of the hash table as more elements are inserted. Compare open addressing with chaining.

4 For the hash table with open addressing, modify the code of `insert()` to use quadratic probing. Illustrate with an example, choosing appropriate table size and sequence of insertions, how quadratic probing may reduce the effect of clustering.