the Stack

1. The Stack Abstract Data Type
   - LIFO, UML class diagram, stack ADT
   - using the STL stack
2. An Application: Test Expressions
   - are parentheses balanced?
   - an algorithm which uses a stack
3. Stack Implementations
   - adapting the STL vector class
   - adapting the STL list class

MCS 360 Lecture 13
Introduction to Data Structures
Jan Verschelde, 12 February 2020

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stacks

A stack is a LIFO (Last In First Out) sequence:
we can only get (or pop) the element on top.

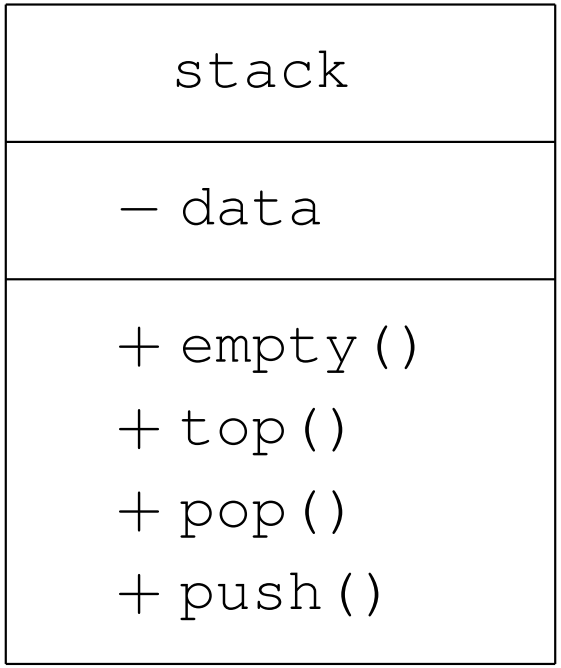
Example: a pile of books, stacked on top of each other.

Despite this restriction, many applications:

- parsing expressions,
- store information about function calls.

In the STL, a stack is an *adapter*:
either a vector or a list implements a stack.

UML class diagram



stack ADT

```
abstract <typename T> stack;
/* a stack is a sequence of elements,
   stored as Last In First Out (LIFO) */

abstract bool empty ( stack s );
postcondition: empty(s)
   == true if s is empty,
   == false if s is not empty;

abstract T top ( stack s );
precondition: not empty(s);
postcondition: top(s) is the top element of s;
```

stack ADT continued: push and pop

```
abstract void push ( stack s, T e );
postcondition: push(s,e); top(s) == e;

abstract void pop ( stack s );
precondition: not empty(s);
postcondition: top(s) is removed from s;
```

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using the STL stack – counting down

Our first program with an STL stack does the following:

1. push given numbers to the stack, until zero is entered;
2. pop stored numbers from the stack, until empty.

```
$ use_stl_stack
give an integer (0 to stop) : 1
give an integer (0 to stop) : 2
give an integer (0 to stop) : 3
give an integer (0 to stop) : 0
popped 3
popped 2
popped 1
$
```

A stack is a natural data structure to reverse the order in any sequence.

computing the binary expansion of a number

Exercise 1: (recall exercise 2 of L-1)

Write a C++ program which prompts the user
for a positive integer number.

The program writes the input number and
prints the bits in the binary decomposition of the number,
in the correct order, printing the most significant bit first.

A session with the program could go as follows:

```
Give a number : 360
The bits in 360 : 1 0 1 1 0 1 0 0 0.
```

using the STL stack

```
#include <iostream>
#include <stack>
using namespace std;

int main()
{
   stack<int> s;
   int e;

   do
   {
      cout << "give an integer (0 to stop) : ";
      cin >> e;
      if(e <= 0) break;
      s.push(e);
   }
   while(true);
```

`top` and `pop`

```
while(!s.empty())
{
   int e = s.top();
   s.pop();
   cout << "popped " << e << endl;
}
```

Members of `stack<T>` class:

- `T top() const` : returns the top of the stack,
- `void pop()` : removes the top element.

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problem statement

Given an expression `(w*(x+y) / z - (p/(9 -8)))`,
test if every closing bracket `)` matches an opener `(`.

Simple counting algorithm:

- $+1$ if encounter opener `(`, and
- -1 if encounter closer `)`.

Expression is balanced if final count equals zero.

Harder if different type of brackets, braces, and parentheses can be used, e.g.: `(w*[x+y] / z - [p/{9 -8}])`.

running the program

```
$ match_brackets
give an expression : (w*[x+y] / z - [p/{9 - 8}])
checking "(w*[x+y] / z - [p/{9 - 8}])" ...
pushed (
pushed [
] matches [ at 7
popped [
pushed [
pushed {
} matches { at 24
popped {
] matches [ at 25
popped [
) matches ( at 26
popped (
parenthesis in "(w*[x+y] / z - [p/{9 - 8}])" \
   are balanced
$
```

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an algorithm which uses a stack

A stack stores all opening brackets:

- we push every opening bracket,
- for every matching closing bracket, we pop.

Outline of the algorithm:

For every character c in a given expression:
if c is '(', '{', or '[' then
push c to a stack
else if c is ')', '}', or ']' then
if top of stack matches c then
pop the stack
else
break out of loop: parenthesis unbalanced.

Careful: stack may be empty.

data and variables

```
#include <iostream>
#include <string>
#include <stack>

using namespace std;

int main()
{
   string expression;

   cout << "give an expression : ";
   getline(cin,expression,'\n');

   stack<char> brackets; // stack of brackets
   const string opening_brackets = "({[";
   const string closing_brackets = ")}]";
```

Positions of corresponding opening/closing brackets match.

recall the `find`

For any string `s` and character `c`:

`s.find(c)` either

- returns `string::npos` if `c` does not occur in `s`,

or

- returns the first index `k` for which `s[k] == c`.

We use `find` with `c == expression[i]`
on `s == opening_brackets`
or `s == closing_brackets`.

pushing and popping brackets

```
bool balanced = true;

for(int i=0; i<expression.size(); i++)
   if(opening_brackets.find(expression[i])
      != string::npos) // push opening bracket
      brackets.push(expression[i]);
   else if(closing_brackets.find(expression[i])
            != string::npos)
   {  // Does top of brackets match closing bracket?
      int k = closing_brackets.find(expression[i]);
      if(brackets.empty())
      {
         cout << "no matching opening bracket for "
              << expression[i] << " at " << i << endl;
         balanced = false; break;
      }
```

Does top of bracket match closing bracket?

As we have

- `closing_brackets.find(expression[i]) != string::npos)`
- `int k = closing_brackets.find(expression[i]);`
- `!brackets.empty()`

we can examing the top of the stack of brackets:

```
char c = brackets.top();
if(c != opening_brackets[k]) // match?
{  // if not at same position, then no match
   cout << "unbalanced parenthesis : "
        << c << " != " << expression[i]
        << " at " << i << endl;
   balanced = false; break;
}
```

we can pop matching opening bracket

```
      else
      {
         cout << expression[i] << " matches " << c
              << " at " << i << endl;
         brackets.pop();
         cout << "popped " << c << endl;
      }
   } // end of handling closing bracket

cout << "parenthesis in \"" << expression << "\"";
if(!balanced)
   cout << " are not balanced" << endl;
else if(brackets.empty())
   cout << " are balanced" << endl;
else
   cout << " are not balanced, missing closing bracket"
        << endl;
```

an application

Exercise 2: balancing brackets in C++ program

Write a program that prompts the user for the name of a file which contains C++ source code.

The program verifies whether every closing bracket in the code matches the corresponding opening bracket:
every `{`, `(`, and `[` is closed respectively by `}`, `)`, and `]`.

In your program, write a short message for every push and pop that happens during the execution of the program.

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adapting STL vectors

A stack is said to be an *adapter* class:
we adapt a sequential container, get a stack implementation providing another interface to vector or list.

A dictionary between STL stack and vector,
for any item `t` of type `T`:

`stack<T> s`	`vector<T> v`
`s.push(t)`	`v.push_back(t)`
`if(!s.empty())`	`if(!v.empty())`
`t = s.top()`	`t = v[v.size()-1]`
`t = s.top()`	`t = v.back()`
`s.pop()`	`v.pop_back()`

STL vector as stack

```
#include <iostream>
#include <vector>
using namespace std;

int main()
{
   vector<int> s;
   int e;

   do
   {
      cout << "give an integer (0 to stop) : ";
      cin >> e;
      if(e <= 0) break;
      s.push_back(e);
   }
   while(true);
```

top and pop

```
while(!s.empty())
{
   int e = s[s.size()-1];
   cout << "popped " << e
        << " = " << s.back() << endl;
   s.pop_back();
}
```

A stack is a natural data structure to reverse the order in any sequence.

Application: check if word is a palindrome.
A palindrome can be read forward and backward, some examples: dad, testset, racecar.

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adapting STL lists

As an alternative to adapting the STL vector class,
we can implement a stack adapting the STL list class.

A dictionary between STL stack and list,
for any item `t` of type `T`:

`stack<T> s`	`List<T> L`
`s.push(t)`	`L.push_back(t)`
`if(!s.empty())`	`if(!L.empty())`
`t = s.top()`	`t = L.back()`
`s.pop()`	`L.pop_back()`

STL list as stack

```
#include <iostream>
#include <list>
using namespace std;

int main()
{
   list<int> s;
   int e;

   do
   {
      cout << "give an integer (0 to stop) : ";
      cin >> e;
      if(e <= 0) break;
      s.push_back(e);
   }
   while(true);
```

top and pop

```
while(!s.empty())
{
   int e = s.back();
   s.pop_back();
   cout << "popped " << e << endl;
}
```

Observe that the uniform naming of methods in STL leads to another type of generic programming:
in the description of the algorithm, we may declare `s`
as `vector<T> s`, or as `list<T> s`.

Summary + Additional Exercises

Started Chapter 5 on *Stacks*. Although simpler than vectors or lists, some algorithms reduce to mere loops with a stack.

Additional Exercises:

3. Write code using a stack to test if a string, given by the user, is a palindrome.
4. The phrase "`murder for a jar of red rum`" is a palindrome. Adjust the code of the first exercise to ignore spaces.
5. Adjust the test for balanced parentheses to match single (left ‘ and right ’) and double quotes ".
6. Use a stack to compute the value of a number given as string in some given basis (< 10). For example: "537" in octal (base 8) evaluates to $7 + 3 \times 8 + 5 \times 8^2$.