

NAME :

The exam is closed book, no notes and no computer.  
 All your answers to the questions below must be submitted on paper.  
 Write your name on this sheet and submit it with your answers.  
 Please do not ask questions during the exam.

1. Schröder numbers can be defined as

$$R_0 = 0 \quad \text{and for } n \geq 1 : \quad R_n = R_{n-1} + \sum_{k=0}^{n-1} R_k R_{n-1-k}.$$

- (a) Explain why the application of the definition in a recursive function to compute  $R_n$  will be wasteful. In your answer, write complete sentences.
  - (b) Write an efficient recursive function `R` to compute  $R_n$  using memoization.
2. Consider a vector of strings, which holds the names of fruit items.  
 For example: "apple", "banana", "lemon", "grape", "orange".

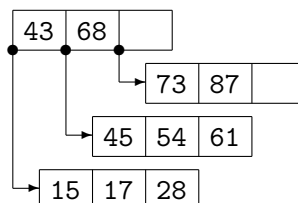
Write a C++ function to show all combinations of  $k$  items, for  $k$  between 1 and the size of the vector. The function writes all combinations to screen, one line per combination. For the example above, if  $k$  equals 3, then the output starts with

```
apple banana lemon
apple banana grape
apple banana orange
apple lemon grape
etc. ...
```

Observe that every fruit item occurs only once. Your function should work for any size of the vector of strings and for any  $k$  between 1 and the size of the vector.

- (a) Write the prototype of your function and document all its parameters.
  - (b) Give the definition of your function.
3. Create a Huffman code for the word "bubblegum".  
 Write all stages in the creation of the code.
4. Consider the numbers 63, 18, 35, 13, 50, 24, 42. Sort the numbers in decreasing order (largest number first, smallest number last) using a heap (or priority queue).  
 Show all stages in the evolution of the heap.

5. Consider the 4-tree:



- (a) Draw the equivalent red-black tree to the 4-tree above.
- (b) Draw the 4-tree after inserting 56 in the 4-tree above.