

NAME :

The exam is closed book, no notes and no computer.

All your answers to the questions below must be submitted on paper.

Write your name on this sheet and submit it with your answers.

Please do not ask questions during the exam.

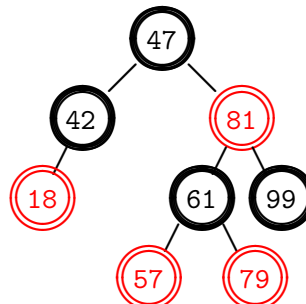
- The Catalan numbers C_n can be defined as $C_0 = 1$ and for $n > 0$: $C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}$.
 - Explain why the application of the definition in a recursive function to compute C_n will be wasteful. In your answer, write complete sentences.
 - Write an efficient recursive function `C` to compute C_n using memoization.
- Consider a vector of strings, which holds the names of vegetables
 For example: "cucumber", "lettuce", "olive", "onion", "spinach".
 Write a C++ function to show all combinations of k vegetables, for k between 1 and the size of the vector. The function writes all combinations to screen, one line per combination.
 For the example above, if k equals 3, then the output starts with

```
cucumber lettuce olive
cucumber lettuce onion
cucumber lettuce spinach
cucumber olive onion
etc. ...
```

Observe that every vegetable occurs only once. Your function should work for any size of the vector of strings and for any k between 1 and the size of the vector.

- Write the prototype of your function and document all its parameters.
 - Give the definition of your function.
- Create a Huffman code for the word "messenger".
 Write all stages in the creation of the code.
 - Consider the numbers 80, 51, 53, 96, 44, 44, 70, 41, 75, 36, 33, 55, 23, 12, 67. Sort the numbers in increasing order (smallest number first, largest number last) using quicksort. Show all stages in the evolution of the sort.

- Consider the red-black tree:
Red nodes have hollow rings.



- Draw the equivalent 2-3-4 tree to the red-black tree above.
- Draw the red-black tree after inserting 50 in the red-black tree above.