Second Review for the Final Exam

1. The Final Exam
   - on Friday 9 December from 10:30am to 12:30pm
2. Some Questions
   - the method of undetermined coefficients
   - Richardson extrapolation
   - linear systems
   - shooting to solve boundary value problems
   - finite differences

MCS 471 Lecture 42
Numerical Analysis
Jan Verschelde, 30 November 2022

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The Final Exam

- Friday 9 December, from 10:30am to 12:30pm, online.
- To participate to the final exam, you must send an email to `janv@uic.edu` before 5pm on Thursday 8 December.
- If you do not send that email, then you will not receive the questions and your final grade will be an F.
- The exam must be solved individually.
  Submitting materials retrieved from the internet is plagiarism.
- Solutions must be in a Jupyter notebook, with a Julia kernel.
- Answers must be submitted before or at 12:30pm.
- Submit to gradescope.

focus of this review

- The focus of this review is on lectures 24 to 35.
  1. Differentiation and Integration
  2. Initial Value Problems
  3. Boundary Value Problems
- The focus is on numerical analysis concepts, not on Julia programming.
- Questions on this review are representative, but the list is by no means exhaustive.
- Please review the homework problems.
- Consider also the second review (Lecture 36), and all versions of the second midterm exam.

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1. the method of undetermined coefficients

The formula $y_{n+1} = y_n + \frac{1}{12} h \left(5 f_{n-2} - 16 f_{n-1} + 23 f_n \right)$

is used to solve $\frac{dy}{dx} = f(x, y(x))$.

Use the method of undetermined coefficients to derive this formula.

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2. Richardson extrapolation

Consider $f(x) = \tan(x)$.
Suppose we want to compute the derivative $f'(0.5)$.

1. Compute forward differences for $h = 1/16, 1/32, 1/64, 1/128$.
2. Apply extrapolation on the computed forward differences.
3. How many decimal places in your answer are correct?
   Justify your estimate.

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3. linear systems

Consider the differential equation

$$\frac{d^4y}{dt^4} - 1.833\frac{d^3y}{dt^3} + 1.500\frac{d^2y}{dt^2} - 5.833\text{e-}01\frac{dy}{dt} + 8.333\text{e-}02y = t,$$

for some initial conditions $y(0) = A$, $y'(0) = B$, $y''(0) = C$, $y'''(0) = D$.

- What is the long term behaviour of the solution?
  Is the solution decaying, oscillating, or diverging?
- Justify your answer.

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4. shooting

Consider the boundary value problem

$$y'' = 18y^2, \quad y(1) = 1/3, \quad y(2) = 1/12.$$

1. Apply `SciPy.integrate.RK45`, once with $y'(1) = -1$, and once with $y'(1) = 0$.
   Write the two values for $y(2)$ at the end with 4 decimal places.
2. Use the two previously obtained values for $y(2)$, to compute the next guess for $y'(1)$.
3. Apply quadratic extrapolation with the three previously obtained values for $y(2)$ to compute the next guess for $y'(1)$.

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5. finite differences

Consider the boundary value problem

$$y'' + xy' = x^2, \quad y'(0) = 1, \quad y'(1) = -1.$$

Apply the finite difference method to this problem.

1. Setup the linear system for $h = 0.2$.
2. How small should h be for an accuracy of 6 decimal places? Justify your answer.