First Review for the Final Exam

1. The Final Exam
   - on Friday 9 December from 10:30am to 12:30pm
2. Some Questions
   - floating-point numbers
   - cost and convergence of the golden section search
   - condition number of a matrix
   - Newton interpolation
   - Gram-Schmidt orthogonalization

MCS 471 Lecture 41
Numerical Analysis
Jan Verschelde, 28 November 2022

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The Final Exam

- Friday 9 December, from 10:30am to 12:30pm, online.
- To participate to the final exam, you must send an email to `janv@uic.edu` before 5pm on Thursday 8 December.
- If you do not send that email, then you will not receive the questions and your final grade will be an F.
- The exam must be solved individually.
  Submitting materials retrieved from the internet is plagiarism.
- Solutions must be in a Jupyter notebook, with a Julia kernel.
- Answers must be submitted before or at 12:30pm.
- Submit to gradescope.

focus of this review

- The focus of this review is on lectures 1 to 21.
  1. Root Finding
  2. Numerical Linear Algebra
  3. Interpolation & Fitting Data
- The focus is on numerical analysis concepts, not on Julia programming.
- Questions on this review are representative, but the list is by no means exhaustive.
- Please review the quizzes and homework problems.
- Consider also the first review (Lecture 22), and all versions of the first midterm exam.

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1. floating-point numbers

Consider a floating-point system with 3 places in the fraction and base 10, with exponents ranging between -16 and $+16$.

1. What is the machine precision in this number system?
2. Compute $3.12 - 3.19$ in this number system.
3. Give three numbers a, b, and c for which $(a+b)+c \neq a+(b+c)$.

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2. cost and convergence of the golden section search

Consider $f(x) = 2 + \cos(x)$, for $x \in [3, 4]$.
Suppose we want to compute the minimum of $f(x)$ over $[3, 4]$.

1. Does the golden section search apply to this problem? Justify.
2. If we want to now the minimum with an accuracy of three decimal places, how many function evaluations will be needed?
   Justify your answer.
3. Verify your calculation of the function evaluations with a run of the code for the golden section search.

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3. condition number of a matrix

Let $\overline{\mathbf{x}}$ be a numerically computed solution of the linear system $A\mathbf{x} = \mathbf{b}$, computed with a precision equal to 64 decimal places.
Suppose the condition number of the matrix A is 10^{24}.

How many decimal places in $\overline{\mathbf{x}}$ are correct?
Justify your answer.

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4. Newton interpolation

Consider the polynomial $p(x) = 2x^2 - 5x + 3$
and interpolation points $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$.
Let $f_0 = p(x_0)$, $f_1 = p(x_1)$, $f_2 = p(x_2)$, $f_3 = p(x_3)$.

1. Compute the table of divided differences.
   What do you observe about the size of the last element?
2. Compute the Newton form of the interpolating polynomial.
   Compare the Newton form with $p(x)$ and
   explain the outcome of your comparison.

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5. Gram-Schmidt orthogonalization

Apply the method of Gram-Schmidt to compute an orthonormal basis for the space spanned by the monomials 1, x, and x^2, using for inner product of two polynomials f and g the integral of the product $f \cdot g$ over $[-1, +1]$.