Review for the Second Midterm Exam

1. The Second Midterm Exam
   - on Wednesday 16 November at 10am
   - skipping policy
2. Some Questions
   - the method of undetermined coefficients
   - Richardson extrapolation and Romberg integration
   - solving initial value problems
   - shooting methods
   - finite differences and finite elements

MCS 471 Lecture 36
Numerical Analysis
Jan Verschelde, 14 November 2022

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The Second Midterm Exam

- Wednesday 15 November, 10am online.
- You must decide if you take the exam by noon Tuesday (15 Nov), send an email to `janv@uic.edu` if you will do the exam.
- The exam must be solved individually.
  Submitting materials retrieved from the internet is plagiarism.
- Solutions must be in a Jupyter notebook, with a Julia kernel.
- Answers must be submitted before on on 10:50am.
- Submit to gradescope.
- Because of the skipping policy, there is no makeup exam.

Topics on the Second Midterm Exam

- The midterm covers lectures 24 to 35.
  1. Numerical Differentiation and Integration
  2. Initial Value Problems
  3. Boundary Value Problems
- The focus is on numerical analysis concepts, not on Julia programming.
- Questions on this review are representative, but the list is by no means exhaustive.
- Please review the projects and homework problems.

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policy on skipping a midterm exam

Please note the policy on skipping the exam:

If an exam is missed, then greater weight will be placed on the final exam, especially on the material covered on the missing exam.

- What this means is that if you decide not to take one midterm exam, your final exam will count for one hundred points more.
- What it does NOT mean is that you can drop the score of a midterm exam.

If you take the midterm, then your score counts.

Please be prepared when you show up for the exam.

Showing up for the exam means that you email `janv@uic.edu` before noon on Tuesday 15 November that you will take the exam.

Without sending an email you will not receive the questions.

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1. the method of undetermined coefficients

Consider the quadrature rule

$$\int_0^h f(x)dx \approx w_0 f(x_0) + w_1 f(x_1), \quad h > 0.$$

1. Set up the system of conditions on the weights w_0, w_1 and nodes x_0, x_1 to obtain the highest possible degree of precision.
2. What is the degree of precision attained by this rule? Justify your answer.

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2. Richardson extrapolation

Consider $f(x) = \exp(x)$.
Suppose we want to compute the derivative $f'(2.0)$.

1. Compute forward differences for $h = 1, 0.5, 0.25, 0.125$.
2. Apply extrapolation on the computed forward differences.
3. How many decimal places in your answer are correct? Justify your estimate.

3. Romberg integration

1. Approximate $\int_1^2 e^x dx$ using the composite Trapezoidal rule for n intervals, for $n = 1, 2, 4, 8$.
2. Apply Romberg integration to improve the approximation.
3. How many decimal places in your answer are correct? Justify your estimate.

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4. higher order equations

Consider the initial value problem

$$y'''(t) + 0.647y''(t) + 0.435y'(t) + 0.412y(t) = 0,$$

with initial values $y(0) = 1$, $y'(0) = 0$, and $y''(0) = 0$.

1. Rewrite this third-order differential equation as a system of first-order differential equations.
2. The roots of $x^3 + 0.647x^2 + 0.435x + 0.412$, given with 3 significant decimal places are -0.773, and $0.0630 \pm 0.727i$, $i = \sqrt{-1}$.

   Knowing the roots of this polynomial (which corresponds to the differential equation), what can you write about the behavior of the solution $y(t)$?

   In particular, does $y(t)$ converge or diverge as t goes to infinity?

5. stability of a method

Consider the implicit two step method

$$y_{n+1} = \frac{4}{3}y_n - \frac{1}{3}y_{n-1} + \frac{2}{3}hf_{n+1}, \quad n = 0, 1, \ldots$$

to solve an initial value problem.

1. Is this method stable? Justify.
2. If stable is this method weakly or strongly stable? Justify.

6. predictor-corrector methods

Suppose we want to solve an initial value problem

$$\frac{dy}{dx} = f(x, y), \quad y(0) = y_0,$$

with a predictor-corrector method using four points in each step.

1. Calculate how many times we evaluate f to approximate $y(1.4)$, using the fixed step size $h = 0.2$.
2. Justify your count.

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7. shooting methods

The boundary value problem

$$\frac{d^2y}{dx^2} = 3, \quad y(-1) = 1, \quad y(+1) = 0$$

has as exact solution $y(x) = \frac{3}{2}x^2 - \frac{1}{2}x - 1$.

1. Let $Y_1 = 0$ and $Y_2 = -1$ be two guesses for $y'(-1)$, leading respectively to 7 and 5 as values for $y(+1)$. What is your next guess for $y'(-1)$?
2. Without referring to the exact solution, explain why your next guess must be the correct value for $y'(-1)$.

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8. finite differences

Consider the boundary value problem

$$\frac{d^2y}{dx^2} = 8x, \quad y(0) = 1, \quad y(1) = 0.$$

1. Divide $[0, 1]$ into 4 intervals of equal size and apply the method of finite differences to set up the linear system to find approximations for $y(x)$ over $[0, 1]$.
2. What is the accuracy of the solution?
   Justify your answer.

9. finite elements

Consider the boundary value problem

$$\frac{d^2y}{dx^2} = 8x, \quad y(0) = 1, \quad y(1) = 0.$$

1. Divide $[0, 1]$ into 4 intervals of equal size and apply the Galerkin method to set up the linear system to find approximations for $y(x)$ over $[0, 1]$.
2. What is the accuracy of the solution?
   Justify your answer.

10. there is always more ...