

MCS 471: Formula Sheet for Exam II

- Lagrange interpolation: $l_i(x) = \prod_{\substack{j=0 \\ \neq i}}^n \frac{x - x_j}{x_i - x_j}$, $p(x) = \sum_{i=0}^n l_i(x)f_i$.
- Neville interpolation: $p_{i\dots j} = \frac{(x^* - x_j)p_{i\dots j-1} - (x^* - x_i)p_{i+1\dots j}}{x_i - x_j}$.
- Divided differences: $f_{0\dots ji} = \frac{f_{0\dots j-1j} - f_{0\dots j-1i}}{x_j - x_i}$
 $p(x) = f_0 + f_{01}(x - x_0) + f_{012}(x - x_0)(x - x_1) + \dots + f_{012\dots n}(x - x_0)(x - x_1) \dots (x - x_{n-1})$.
- Interpolation error: $E(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x - x_0)(x - x_1) \dots (x - x_n)$.
- Chebyshev polynomials: $T_n(x) = \cos(n \arccos(x))$
 $T_0(x) = 1$, $T_1(x) = x$, $T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$, $n > 0$.
- Taylor: $f(x + h) = f(x) + hf'(x) + h^2 \frac{f''(x)}{2!} + h^3 \frac{f^{(3)}(x)}{3!} + O(h^4)$.
Maclaurin: $f(0 + h) = f(0) + hf'(0) + h^2 \frac{f''(0)}{2!} + h^3 \frac{f^{(3)}(0)}{3!} + O(h^4)$.
- $If(x) = f(x)$, $Df(x) = \frac{\partial f}{\partial x}$, $D = \frac{\partial}{\partial x}$, $Ef(x) = f(x + h)$, $E^{-1}f(x) = f(x - h)$.
 $\Delta f(x) = f(x + h) - f(x)$, $\Delta = E - I$. $\nabla f(x) = f(x) - f(x - h)$, $\nabla = I - E^{-1}$.
 $\delta f(x) = f(x + h) - f(x - h)$, $\delta = E - E^{-1}$.
- $D = \frac{1}{h} \left(\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \frac{\Delta^5}{5} - \dots \right)$ $D = \frac{1}{h} \left(\nabla + \frac{\nabla^2}{2} + \frac{\nabla^3}{3} + \frac{\nabla^4}{4} + \frac{\nabla^5}{5} + \dots \right)$
- Richardson extrapolation ($0 < r < 1$):
 $\Delta f(x, h) = \frac{1}{h} \Delta f(x)$ $\Delta f(x, h, rh, \dots, r^n h) = \frac{\Delta f(x, h, rh, \dots, r^{n-1} h)r^n - \Delta f(x, rh, r^2 h, \dots, r^n h)}{r^n - 1}$
 $\delta f(x, h) = \frac{1}{2h} \delta f(x)$ $\delta f(x, h, rh, \dots, r^n h) = \frac{\delta f(x, h, rh, \dots, r^{n-1} h)r^{2n} - \delta f(x, rh, r^2 h, \dots, r^n h)}{r^{2n} - 1}$
- Trapezoidal rule: $\int_a^b f(x)dx = \frac{f(a) + f(b)}{2}(b - a)$,
composite Trapezoidal rule: $T(h) = \frac{h}{2}(f(a) + f(b)) + h \sum_{k=1}^{n-1} f(a + kh)$, $h = \frac{b-a}{n}$.
Romberg integration: $T[i][j] = \frac{T[i][j-1]2^{2j} - T[i-1][j-1]}{2^{2j} - 1}$, $T[i][0] = T\left(\frac{h}{2^i}\right)$.
- Fourier series: $f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(2\pi kt) + b_k \sin(2\pi kt) = \sum_{k=-\infty}^{\infty} c_k e^{i2\pi kt}$.
 $a_k = \frac{1}{2} \int_0^1 f(t) \cos(2\pi kt) dt$, $b_k = \frac{1}{2} \int_0^1 f(t) \sin(2\pi kt) dt$
 $c_k = \frac{1}{2}(a_k - ib_k)$, $c_{-k} = \frac{1}{2}(a_k + ib_k)$.