FINAL EXAM MCS 471 Jan Verschelde Monday 29 April 2002

NAME : Type of Calculator :

You may use a calculator. Formula sheets are attached.

page	1	2	3	4	5	6	7	total
points								
maximum	25	35	25	30	25	30	30	200

1. Consider the representation of floating-point numbers with base 10 and 2 digits in the fraction part. The values for the exponents are between -10 and $+10$.

   (a) What is the machine precision is this number system?

   (b) Represent the numbers 319 and 284 as floating-point numbers.
   Illustrate the calculation of $319 + 284$ and $319 - 284$. Use rounding everywhere.

   /15

2. What is error propagation? Give an illustration on a method we have seen.

   /10

3. Consider $f(x) = x^2 - 3x$. Apply three steps of the golden section search method to find the minimum of $f(x)$ in the interval $[0, 1]$.

   Write the values for a, b, x_1, x_2, $f(x_1)$, and $f(x_2)$ in the table (4 decimal places):

step	a	b	x_1	x_2	$f(x_1)$	$f(x_2)$
0	0.000	1.000				
1						
2						

   /15

4. Consider the linear system $A\mathbf{x} = \mathbf{b}$. The condition number of the matrix A is 10^9. Assuming the relative error on A is of magnitude 10^{-16}, and using exact arithmetic, how many decimal places in the answer can we trust? Justify your answer.

   /10

5. Suppose you are given a **nonlinear** boundary-value problem. Which method would you choose to solve it: shooting or finite differences? Justify your choice.

   /10

6. Consider the matrix $A = \begin{bmatrix} -1.000 & 4.000 & -1.000 \\ 1.000 & 2.000 & 4.000 \\ 3.000 & -3.000 & 4.000 \end{bmatrix}$.

(a) Compute the LU decomposition of A with partial pivoting.

Calculate with four decimal places, using rounding: write the answer of every step rounded to four decimal places, and use the rounded number in the calculations of the next step.

(b) What is the determinant of A?

/25

7. We sampled a function f at $x = 0, 1, 2, 3$ and obtained the function values $f(0) = -6$, $f(1) = -5$, $f(2) = -2$, and $f(3) = 3$. Compute the Newton form of the polynomial which interpolates the data $(i, f(i))$, $i = 0, 1, 2, 3$. /15

8. The Maclaurin expansion of $\arcsin(x)$ is

$$\arcsin(x) = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \frac{35}{1152}x^9 + O(x^{10})$$

Use this Maclaurin expansion to construct a Padé approximation for $\arcsin(x)$ as a quotient of two quadrics. /15

9. Consider a function whose values are tabulated below:

x	$f(x)$
0.000	0.0000000000
0.125	0.1256551366
0.250	0.2553419212
0.375	0.3936265759
0.500	0.5463024898
0.625	0.7214844410
0.750	0.9315964599
0.875	1.197421629
1.000	1.557407725

(a) Compute the most accurate approximation for $f'(0.5)$.

(b) How many decimal places in your answer are correct? Justify.

/25

10. Apply the method of underdetermined coefficients to derive an Adams-Bashforth formula which uses three function evaluations. /15

11. Suppose we want to solve the initial-value problem $y' = f(x, y)$, $y(0) = 1$, with a predictor-corrector method using four points in each step. Calculate how many times we evaluate f to approximate $y(0.7)$, using step size $h = 0.1$. /15

12. Consider the boundary-value problem $y'' + y = 3x^2$, $y(0) = 1$, and $y'(1) = 0$.

Give the linear system one has to solve in the method of finite differences with $h = 0.2$. Be considerate for the mixed boundary conditions. /15

13. Consider the boundary-value problem $y'' + y' = x$, with $y(0) = 2$ and $y(1) = 5$.

With our first guess $y'(0) = 1$ we find 0.735758 at $x = 1$. Our second guess $y'(0) = 3$ yields 1.47152 at $x = 1$. What is your next guess for $y'(0)$ in the shooting method? /15