MCS 471: Review of Chapters 6 and 7

The questions below are meant to stimulate the preparation of the final exam, concerning chapters 6 and 7. They cover some of the most important but not all topics. You must review all course materials: textbook, homework exercises, lecture notes (also consult the course web site), and quizzes. Also please review the computer projects.

1. Apply the method of underdetermined coefficients to derive an Adams-Moulton formula which uses three function evaluations. What is the order of the local error (in terms of the step size h) of the method you just derived? Justify your answer.

2. Consider the equation $y'' = x + y$, with $y(0) = 1$ and $y'(0) = 2$. Apply a predictor-corrector method, using three points in each step to approximate $y(0.5)$, with step size $h = 0.1$.

3. Derive a bound on the step size h so that Euler's method converges on the problem $y' = \lambda y$, $y(0) = 1$, for $\lambda < 0$. Can you also derive a bound on h for the modified Euler's method?

4. Consider the following boundary-value problem:

$$y'' - 2xy = x^3, \quad y(0) = 1, \quad y(1) = -1.$$

   With the shooting method we translate this problem into initial-value problems.

   (a) With our first guess $y'(0) = 1$ we find 3.12767 at $x = 1$.
   Our second guess $y'(0) = -1$ yields -0.320365 at $x = 1$.
   What is your next guess for $y'(0)$ in the shooting method?

   (b) Suppose we modify the differential equation above into $y'' - 2y^2 = x^3$. Is the modified problem easier or harder than the original one? Justify your answer.

   (c) Suppose, instead of the conditions $y(0) = 1$ and $y(1) = -1$ we have mixed boundary conditions: $y'(0) = 1$ and $y(1) = -1$ for the original ODE above. Describe, in general (but in detail), how the shooting method works to solve this problem.

5. Consider the boundary-value problem

$$y'' + xy' = x^2, \quad y(0) = 1, \quad y(1) = -1.$$

   With finite-difference approximations we translate this problem into a linear system.

   (a) Give the linear system one has to solve for $h = 0.2$.

   (b) Suppose, instead of the given boundary conditions, we are given $y'(0) = 1$ and $y'(1) = 1$. Given the linear system one has to solve for $h = 0.2$.

6. The equation below depends on a parameter k:

$$y'' - 3y' + 2k^2 y = 0, \quad y(0) = 0, \quad y(1) = 0.$$

   With finite-difference approximations we translate this problem into an eigenvalue problem.

   (a) Give the eigenvalue problem one has to solve for h = 0.2.

   (b) Explain how to approximate the eigenvector whose corresponding eigenvalue lies closest to 1.

FINAL EXAM is in BH 0308 on Monday 8 December 2003, from 1 till 3PM.