MCS 471: Formula Sheet for Exam II

1. Lagrange interpolation: $l_i(x) = \displaystyle\prod_{\substack{j=0\\ \neq i}}^{n} \frac{x - x_j}{x_i - x_j}, \quad p(x) = \sum_{i=0}^{n} l_i(x) f_i.$

2. Neville interpolation: $p_{i\ldots j} = \dfrac{(x^* - x_j)p_{i\ldots j-1} - (x^* - x_i)p_{i+1\ldots j}}{x_i - x_j}.$

3. Divided differences: $f_{0\ldots ji} = \frac{f_{0\ldots j-1j} - f_{0\ldots j-1i}}{x_j - x_i}$
   $p(x) = f_0 + f_{01}(x-x_0) + f_{012}(x-x_0)(x-x_1) + \cdots + f_{012\ldots n}(x-x_0)(x-x_1)\cdots(x-x_{n-1}).$

4. Interpolation error: $E(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x - x_0)(x - x_1)\cdots(x - x_n).$

5. Chebyshev polynomials: $T_n(x) = \cos(n \arccos(x))$
   $T_0(x) = 1, \quad T_1(x) = x, \quad T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \quad n > 0.$

6. Taylor: $f(x+h) = f(x) + hf'(x) + h^2\frac{f''(x)}{2!} + h^3\frac{f^{(3)}(x)}{3!} + O(h^4).$
   Maclaurin: $f(0+h) = f(0) + hf'(0) + h^2\frac{f''(0)}{2!} + h^3\frac{f^{(3)}(0)}{3!} + O(h^4).$

7. $If(x) = f(x), \quad Df(x) = \frac{\partial f}{\partial x}, D = \frac{\partial}{\partial x}, \quad Ef(x) = f(x+h), E^{-1}f(x) = f(x-h).$
   $\Delta f(x) = f(x+h) - f(x), \Delta = E - I.$ $\nabla f(x) = f(x) - f(x-h), \nabla = I - E^{-1}.$
   $\delta f(x) = f(x+h) - f(x-h), \delta = E - E^{-1}.$

8. $D = \frac{1}{h}\left(\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \frac{\Delta^5}{5} - \cdots\right) \quad D = \frac{1}{h}\left(\nabla + \frac{\nabla^2}{2} + \frac{\nabla^3}{3} + \frac{\nabla^4}{4} + \frac{\nabla^5}{5} + \cdots\right)$

9. Richardson extrapolation ($0 < r < 1$):
   $\Delta f(x,h) = \frac{1}{h}\Delta f(x) \quad \Delta f(x,h,rh,\ldots,r^n h) = \frac{\Delta f(x,h,rh,\ldots,r^{n-1}h)r^n - \Delta f(x,rh,r^2h,\ldots,r^n h)}{r^n - 1}$
   $\delta f(x,h) = \frac{1}{2h}\delta f(x) \quad \delta f(x,h,rh,\ldots,r^n h) = \frac{\delta f(x,h,rh,\ldots,r^{n-1}h)r^{2n} - \delta f(x,rh,r^2h,\ldots,r^n h)}{r^{2n} - 1}$

10. Trapezoidal rule: $\displaystyle\int_a^b f(x)dx = \frac{f(a) + f(b)}{2}(b - a),$
    composite Trapezoidal rule: $T(h) = \dfrac{h}{2}(f(a) + f(b)) + h\displaystyle\sum_{k=1}^{n-1} f(a + kh), \quad h = \frac{b-a}{n}.$
    Romberg integration: $T[i][j] = \frac{T[i][j-1]2^{2j} - T[i-1][j-1]}{2^{2j} - 1}, \quad T[i][0] = T\left(\frac{h}{2^i}\right).$

11. Fourier series: $F(t) = \dfrac{a_0}{2} + \displaystyle\sum_{k=1}^{\infty} a_k \cos(\pi kt) + b_k \sin(\pi kt) = \sum_{k=-\infty}^{\infty} c_k e^{i\pi kt}.$
    $a_k = \displaystyle\int_{-1}^{+1} f(t)\cos(2\pi kt)dt, k > 0, \quad a_0 = \int_{-1}^{+1} f(t)dt, \quad b_k = \int_{-1}^{+1} f(t)\sin(\pi kt)dt$
    $c_k = \frac{1}{2}(a_k - ib_k), \quad c_{-k} = \frac{1}{2}(a_k + ib_k).$