

MCS 471 Project Five: Solving Initial Value Problems

The goal of the project is to investigate numerical aspects of solving initial value problems. We will use Maple in our investigations. To start working on the project, download the companion Maple worksheet from the web pages for this course.

0. Numerically Solving IVP's in Maple

We will use `dsolve`. With the option `numeric`, by default a pair of Runge-Kutta method is applied, the so-called Runge-Kutta-Fehlberg method, see page 462 in the 6th edition and page 344 in the 7th edition of our book. Here we illustrate its usage to our test equation:

```
[> ode := diff(y(x),x) = y(x):      # ordinary differential equation
[> ini := y(0) = 1:                  # initial value
[> ivp := {ode,ini}:                 # initial value problem
[> sol := dsolve(ivp,y(x),numeric);
```

On return is a procedure, which applies the Runge-Kutta-Fehlberg method. As we are usually only interested in the solution trajectory $y(x)$, we create a function, selecting only what we really need from the solution:

```
[> y := x -> rhs(sol(x)[2]);        # create the function y(x)
[> plot(y,0..1);                    # plot the solution y(x) for x = 0..1
```

1. A Planar Orbit

In astronomy we consider the planar motion of two masses $m_1, m_2 = 1 - m_1$, and a third body of negligible mass moving in the same plane. The motion is governed by the equations

$$u_1''(t) = u_1(t) + 2u_2'(t) - \frac{m_2(u_1(t) + m_1)}{d_1} - \frac{m_1(u_1(t) - m_2)}{d_2}, \quad (1)$$

$$u_2''(t) = u_2(t) - 2u_1'(t) - \frac{m_2 u_2(t)}{d_1} - \frac{m_1 u_2(t)}{d_2}, \quad (2)$$

with

$$d_1 = ((u_1(t) + m_1)^2 + u_2(t)^2)^{3/2}, \quad d_2 = ((u_1(t) - m_2)^2 + u_2(t)^2)^{3/2}, \quad (3)$$

and $m_1 = .012277471, m_2 = 1 - m_1$. The initial values we take are

$$u_1(0) = 0.994, \quad u_2(0) = 0, \quad u_1'(0) = 0, \quad u_2'(0) = -2.00158510637908252240537. \quad (4)$$

Calling the solutions $u_1(t)$ and $u_2(t)$ returned by Maple's `dsolve` `u1` and `u2`, with `plot([u1,u2,0..T])` we plot a periodic orbit, when the appropriate value for `T` is used. The orbit looks like in Figure 1.

Assignment One. We are interested in the period, the number of function values it takes to solve this problem, and the sensitivity of the initial values. In particular, answer the three following questions:

1. What is the minimal range for t to make the plot above with one full orbit?
2. Adding `maxfun = 8000` as last argument to `dsolve` limits the number of function evaluations to 8000. Use this to discover how many function evaluations it takes to plot an entire orbit.
3. How many digits of $u_2'(0) = -2.00158510637908252240537$ are necessary to show the same orbit?

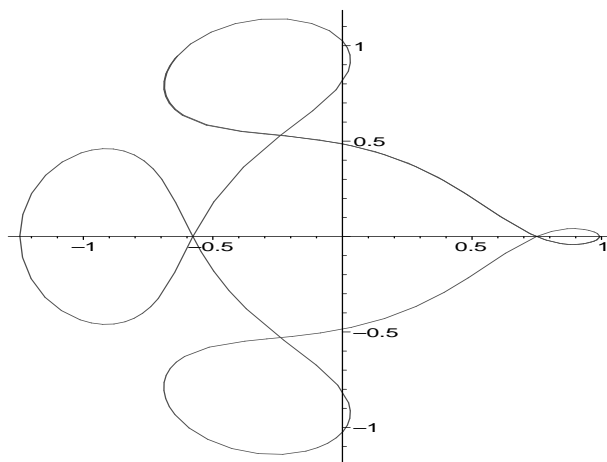


Figure 1: A Planar Orbit

2. A Stiff Problem

Consider the initial value problem

$$y'(t) = \lambda(ty^2(t) - t^{-1}) - t^{-2}, \quad y(1) = 1, \quad \text{for } t \geq 1. \quad (5)$$

Regardless the value of the parameter λ , the exact solution is $y(t) = 1/t$.

Assignment Two. We will investigate the difficulty of this problem for several values of λ .

1. Use **dsolve** with option **numeric** to solve this problem for $\lambda = -5$ for $t \in [1, 2]$. Adjust the righthand-side of **maxfun =** to find the minimal number of function evaluations it takes to reach $t = 2$.
2. Take $\lambda = -500$ and solve the same problem again, for $t \in [1, 2]$. How many function evaluations does it take this time to compute $y(2)$?
3. For both $\lambda = -5$ and $\lambda = -500$, restrict **dsolve** to use only *one* function evaluation (i.e.: **maxfun=1**). How far (in t) do you get in both cases? Interpret your observations. (*Read section 6.6 in the 7th edition and pages 484-486 on Stiff Equations in the 6th edition of the textbook.*)

3. The deadline is Wednesday 27 April 2005 at 10AM

Bring *your* solution to the project to class. The *your* is emphasized to stress that your solution is the result of an *individual* effort. Collaborations are **not** permitted.

Your solution should contain the following:

1. Answers to the questions in the assignments. Please write complete grammatically correct sentences.
2. Tables summarizing the numerical experiments you have done.
3. A print out of the Maple worksheet to show the set up of your calculations may be included *as an appendix*. Suppress long output with colons.

Do not Email me Maple worksheets. Also, summarize your calculations, it is not necessary to print out every single case you computed.

If you have questions or difficulties with the assignments, feel free to come to my office for help.