MCS 471 Project Six: Solving Boundary Value Problems

The goal of the project is to investigate numerical aspects of solving boundary value problems with finite difference methods. We will use MATLAB (or Octave) in our investigations. To start working on the project, download the scripts `meshsys.m` and `mesheig.m` from the web site for this course.

This project offers the opportunity to earn 40 bonus points, which will be added to the grand total, so it make up for points lost on projects, quizzes, midterm exams, or also on the upcoming final exam.

0. Numerically Solving BVP's in MATLAB and Octave

The method of finite differences leads to a linear system, in case of boundary value problems, or to an eigenvalue problem, in case of characteristic value problems. In this project, we will use two small m-files `meshsys.m` and `mesheig.m` to define the data for the boundary value problem and the characteristic value problem respectively.

1. A Boundary Value Problem

Consider $y'' + y = 0$ over $[0, \frac{\pi}{2}]$ with boundary conditions $y(0) = 1$ and $y(\frac{\pi}{2}) = 1$. The exact solution is $y(x) = \cos(x) + \sin(x)$. We use the script `meshsys.m` to set up the linear system for varying values of n, the number of internal nodes in the interval $[0, \frac{\pi}{2}]$. The linear system defined by the script returns values for $y_k \approx y(x_k)$, where $x_k = 0 + kh$, $h = \frac{\pi}{2(n+1)}$.

Assignment One. We want to know how small h (or how large n) should be for the error $\max_{k=1}^{n} |y_k - y(x_k)|$ to be small enough.

Solve the boundary value problem for increasing values of n, for $n = 10, 20, \ldots, 200$. Define the error as the maximal difference between the computed values and the values sampled from the exact solution at the mesh points. Make a table with the errors. Compute $\log(\text{error})/\log(h)$ to see the relationship between the mesh size h and the error.

What is the relation between the error and mesh size h?

2. A Characteristic Value Problem

Consider $y'' + \omega^2 y = 0$ over $[0, 1]$ with boundary conditions $y(0) = 0$ and $y(1) = 0$, for some parameter ω. The exact solution is a family of solutions $y_n(x) = \sin(n\pi x)$, for n any integer number. We use the script `mesheig.m` to define the eigenvalue problem $Ay = \lambda y$, for varying values of n, the number of internal nodes in the interval $[0, 1]$. The components of the eigenvectors give approximations for the solution y_n, for increasing frequencies of the sine function.

Assignment Two. We ask the same question as in the first assignment, i.e.: how large should n be to have an accurate solution? As before, we compute the error as $\max_{k=1}^{n} |y_k - y(x_k)|$.

Solve the characteristic value problem for increasing values of n, for $n = 10, 20, \ldots, 200$. Consider the first eigenvector, with the smallest eigenvalue. Define the error as the maximal difference between the computed values and the values from $\sin(\pi x)$ sampled at the mesh points. Make a table with the errors. Compute $\log(\text{error})/\log(h)$ to see the relationship between the mesh size h and the error.

What is the relation between the error and mesh size h?

3. The deadline is Friday 29 April 2005 at 10AM

Bring *your* solution to the project to class. The *your* is emphasized to stress that your solution is the result of an *individual* effort. Collaborations are **not** permitted.

Your solution should contain the following:

1. Answers to the questions in the assignments. Please write complete grammatically correct sentences.
2. Tables summarizing the numerical experiments you have done.
3. The scripts you used to produce the tables.
4. A print out of the file produced by diary may be included *as an appendix*.

If you have questions or difficulties with the assignments, feel free to come to my office for help.