# review for the midterm exam

1. The Midterm Exam
   - Monday 2 March 2026, at noon

2. Some Questions
   - Taguchi quality control
   - running a simulation
   - solving a recursion
   - a linear programming problem
   - cost benefit analysis
   - supply and demand

MCS 472 Lecture 20
Industrial Math & Computation
Jan Verschelde, 27 February 2026

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# The Midterm Exam

- Monday 2 March, noon online.
- The exam must be solved individually.
  Submitting materials retrieved from the internet is plagiarism.
- Solutions must be in a Jupyter notebook, with a Julia kernel.
- Answers must be submitted before or on 12:50pm, on Monday.
- Submit to gradescope.
- Because of the skipping policy, there is no makeup exam.
- Answers submitted by noon on Wednesday 4 March
  will receive homework credit.

# topics on the midterm exam

The midterm exam covers the first 19 lectures:

1. statistical reasoning and Monte Carlo methods
2. Taguchi quality control
3. analyzing data
4. z-transforms, linear recursions
5. properties of the discrete Fourier transform and the FFT
6. filter design and image processing with the FFT
7. linear programming and regression
8. cost-benefit analysis, present value of future sums
9. microeconomics and macroeconomics

Please review the homework problems.

Have all packages we have covered installed.

Solutions to this review are posted in the posted notebook.

# Julia packages we used

Lecture numbers indicates the first encounter:

- IJulia, Plots, QuadGK (L-1)
- Statistics, StatsKit (L-3)
- DelimitedFiles, DataFrames (L-5)
- SymPy (L-6)
- FFTW (L-10)
- Images (L-11)
- JuMP, GLPK (L-15)
- GLM (L-16)
- FastGaussQuadrature (L-17)
- NLsolve (L-18)

Upgrading to a newer version of Julia may require at least a recompilation, possibly a download, of all packages.

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# fair price for a new medicine?

Suppose a medicine to reduce fever is effective in 80%
after the first dosis and costs \$1.

A temperature reading of more than two degrees above normal
six hours after taking the first dosis requires a visit
to the doctor which costs \$10.

Clinical tests for a new fewer reducing medicine reported a success rate of 95% after taking the first dosis.

1. Under the same circumstances as above, is a cost of \$2 for the new medicine justified? (*Hint:* compute the expected loss of having a fever for both medicines.)
2. What is a fair price for the new medicine, taking into account its higher quality?

At which price would you buy the new medicine?

# computing the expected loss

Suppose a medicine to reduce fever is effective in 80%
after the first dosis and costs \$1.

A temperature reading of more than two degrees above normal
six hours after taking the first dosis requires a visit
to the doctor which costs \$10.

Clinical tests for a new fewer reducing medicine reported a success
rate of 95% after taking the first dosis.

1. The expected loss for the first medicine:

$$0.80 \;\cdot\; \$1 + 0.2 \cdot (\$1 + \$10) = \$3.0.$$

For the new medicine:

$$0.95 \;\cdot\; \$2 + 0.05 \cdot (\$2 + \$10) = \$2.5.$$

The patient is better off with the new medicine.

# the fair price for the new medicine

Suppose a medicine to reduce fever is effective in 80%
after the first dosis and costs \$1.

A temperature reading of more than two degrees above normal
six hours after taking the first dosis requires a visit
to the doctor which costs \$10.

Clinical tests for a new fewer reducing medicine reported a success
rate of 95% after taking the first dosis.

2. For a fair price, we assume the same expected loss.
   Let $x$ be the break even price, with both medicines the same loss.

$$\begin{aligned} 3.0 &= 0.95x + 0.05(x + 10) \\ &= x + 0.5 \\ &\Rightarrow x = \$2.5. \end{aligned}$$

If $x \leq \$2.5$, then we buy the medicine. Otherwise not.

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# running a simulation

A man decides to participate in a 1,000 mile car race.

Because of the rough terrain and excessive speed, tires have a life time normally distributed with an average of 600 miles and standard deviation of 200 miles.

The man starts the race with 4 new tires and 4 spare tires.
Given the expected life span of the tires,
what are the odds to finish the 1,000 mile race?

1. Set up a simulation to decide the odds of to finish the race.
2. Run the simulation 10,000 times and give the probability of reaching the finish.

# code for the simulation

```
function run(N::Int64)
    successes = 0
    for k=1:N
        tires = 600 .+ randn(8)*200
        life = tires[1:4]
        for j=1:4
            life = sort(life)
            life[1] = life[1] + tires[4+j]
        end
        if minimum(life) >= 1000
            successes = successes + 1
        end
    end
    return successes
end
```

# running the simulation

```
run(10000)/10000
```

gives

```
    3729
```

So the probability is about .37

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# a recursion

Suppose we start with an empty savings account.

At the last day of every month \$400 is deposited into the account.

After the first deposit, 20% of the balance of the savings account is withdrawn at the first day of every month.

1. Set up the recursion relation for the balance $B(n)$ at month $n$, after $n$ deposits.
2. Solve the recursion for $B(n)$.
3. In doing so, will we ever get rich? Is there a limit to $B(n)$?

# setup of the recursion

Suppose we start with an empty savings account.

At the last day of every month \$400 is deposited into the account.

After the first deposit, 20% of the balance of the savings account is withdrawn at the first day of every month.

1. Set up the recursion relation for the balance $B(n)$ at month $n$, after $n$ deposits.

$$B(n) = 0.8B(n-1) + 400, \quad B(0) = 0.$$

# solving the recursion

2. Solve the recursion for $B(n)$.

$$B(n) = 0.8B(n-1) + 400, \quad B(0) = 0.$$

We can solve this recursion by substitution.

$$\begin{aligned} B(n) &= \frac{4}{5} B(n-1) + 400 \\ &= \frac{4}{5}\left(\frac{4}{5} B(n-2) + 400\right) + 400 \\ &\vdots \\ &= \left(\frac{4}{5}\right)^n B(0) + 400 \sum_{k=0}^{n-1} \left(\frac{4}{5}\right)^k \end{aligned}$$

# the solution of the recursion

$$B(n) \quad = \quad \left(\frac{4}{5}\right)^n B(0) + 400 \sum_{k=0}^{n-1} \left(\frac{4}{5}\right)^k$$

Now use $B(0) = 0$ and the explicit formula for the geometric sum.

$$400 \sum_{k=0}^{n-1} \left(\frac{4}{5}\right)^k = 400 \frac{\left(\frac{4}{5}\right)^n - 1}{\frac{4}{5} - 1} = 2000 \left(1 - \left(\frac{4}{5}\right)^n\right)$$

3. In doing so, will we ever get rich? Is there a limit to $B(n)$?

We will never get more than $2000.

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# ordering new boats

A boat rental company must order new boats for next summer.
There are three types of boats:

| boat type #seats | storage area | purchase cost | rental price | need for boats |
|---|---|---|---|---|
| 1 | $f_1 = 2 \text{ ft}^2$ | $c_1 = \$122$ | $p_1 = \$5$ | $n_1 = 20$ |
| 2 | $f_2 = 3 \text{ ft}^2$ | $c_2 = \$130$ | $p_2 = \$7$ | $n_2 = 15$ |
| 3 | $f_3 = 4 \text{ ft}^2$ | $c_3 = \$150$ | $p_3 = \$9$ | $n_3 = 10$ |

Boat $i$ requires $f_i$ square feet to store, costs $c_i$ dollars to purchase, and can be rented at $p_i$ dollars a trip. Our constraints are as follows:

1. We need at least $n_i$ boats of type $i$,
2. but have only 400 square feet to store the boats, and
3. our budget is limited to $\$10,000$.

Determine how many boats of each type we should order.

# the linear programming problem

| boat type #seats | storage area | purchase cost | rental price | need for boats |
|---|---|---|---|---|
| 1 | $f_1 = 2 \text{ ft}^2$ | $c_1 = \$122$ | $p_1 = \$5$ | $n_1 = 20$ |
| 2 | $f_2 = 3 \text{ ft}^2$ | $c_2 = \$130$ | $p_2 = \$7$ | $n_2 = 15$ |
| 3 | $f_3 = 4 \text{ ft}^2$ | $c_3 = \$150$ | $p_3 = \$9$ | $n_3 = 10$ |

We need at least $n_i$ boats of type $i$, but have only 400 square feet to store the boats, and our budget is limited to $\$10,000$.

Let $x_i$ be the number of boats of type $i$.

$$\max 5x_1 + 7x_2 + 9x_3$$

$$\text{subject to} \quad \begin{array}{rcl} 2x_1 + 3x_2 + 4x_3 & \leq & 400 \\ 122x_1 + 130x_2 + 150x_3 & \leq & 10000 \\ x_1 & \geq & 20 \\ x_2 & \geq & 15 \\ x_3 & \geq & 10 \end{array}$$

# defining the dataframe for the problem

We are using the Julia optimization package JuMP:

```
using JuMP
import DataFrames
import GLPK
```

and store the data in a dataframe:

```
boats = DataFrames.DataFrame(
    [
        1 2 122 5 20
        2 3 130 7 15
        3 4 150 9 10
    ],
    ["type", "area", "cost", "price", "need"]
)
```

# defining the model

```
model = Model(GLPK.Optimizer)

@variable(model, x[boats.type] >= 0)

@objective(
    model,
    Max,
    sum(boat["price"] * x[boat["type"]]
        for boat in eachrow(boats))
)
```

defines the objective:

$$\max 5x_1 + 7x_2 + 9x_3$$

# adding constraints

The first contraint is on the storage:

```
storage = @expression(
    model,
    sum(boat["area"] * x[boat["type"]]
        for boat in eachrow(boats)),
)
@constraint(model, storage <= 400)
```

The second constraint is on the budget:

```
budget = @expression(
    model,
    sum(boat["cost"] * x[boat["type"]]
        for boat in eachrow(boats)),
)
@constraint(model, budget <= 10000)
```

# adding the need for each type of boat

```
for boat in eachrow(boats)
    @constraint(model,
        x[boat["type"]] >= boat["need"])
end
```

To verify whether we have the model defined correctly, do

```
print(model)
```

See the posted Jupyter notebook.

# solving the problem

```
optimize!(model)
solution_summary(model)
```

prints

```
* Solver : GLPK

* Status
  Termination status : OPTIMAL
  Primal status      : FEASIBLE_POINT
  Dual status        : FEASIBLE_POINT
  Message from the solver:
  "Solution is optimal"

* Candidate solution
  Objective value      : 541.5999999999999
  Objective bound      : Inf
  Dual objective value : 541.6

* Work counters
  Solve time (sec)   : 0.00600
```

# printing the solution

```
for boat in boats.type
    println("x[",boat, "] = ", value(x[boat]))
end
```

shows

```
x[1] = 20.0
x[2] = 15.0
x[3] = 37.4
```

which corresponds to the value of 541.6 of the objective.

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# cost benefit analysis

An investment of \$10,000 will save us \$1,500 each year for the coming eight years.

- Use continuous compounding and a discount rate of 6% to compute the present worth of the savings.
- Is the investment worthwhile?

# the present value of the future savings

We compute the present value of the future savings:

```
julia> s = 1500; r = 0.06;

julia> sum([s*exp(-r*t) for t=1:8])
9247.361701729767
```

Since the present value is less than $10,000,
the investment is not worthwhile.

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# supply and demand

Consider a market with demand $q = 5 + 10/p$ and supply $q = p^2 - 3$.

1. Compute the equilibrium price and the revenue.
2. Suppose the government gives the producer a subsidy of \$1 per item. What is the effect of this subsidy?

   Compute the new supply, equilibrium price, and revenue function.
3. Of every dollar the government spends on subsidy, how much goes to producer, and how much to the consumer?

# equilibrium and revenue

Consider a market with demand $q = 5 + 10/p$ and supply $q = p^2 - 3$.

1. Compute the equilibrium price and the revenue.

Using `NLsolve`:

$$5 + 10/p = p^2 - 3 \quad \Rightarrow \quad p = \$3.32.$$

Then the revenue is

$$R = p \cdot S(p) = \$3.32 \cdot 8.01 = \$26.59.$$

# effect of the subsidy

Consider a market with demand $q = 5 + 10/p$ and supply $q = p^2 - 3$.

2. Suppose the government gives the producer a subsidy of \$1 per item. What is the effect of this subsidy?
Compute the new supply, equilibrium price, and revenue function.

The new supply function is

$$S(p+1) = (p+1)^2 - 3.$$

The new equilibrium price: \$2.47.

The new supply: 9.05.

The new revenue: \$22.35.

# benefit for the consumer

Consider a market with demand $q = 5 + 10/p$ and supply $q = p^2 - 3$.

3. Of every dollar the government spends on subsidy, how much goes to producer, and how much to the consumer?

The benefit of the consumer:

$$\$3.32 - \$2.47 = \$0.85,$$

so the consumer gets 85 cents for the $1 subsidy.

Of the $1 subsidy, 15 cents goes to the producer.