proposals for the second project

1. The Second Project
   - presentations in week ten
2. Topics
   - the discrete Fourier transform
   - linear programming
   - regression
   - least squares Chebyshev and Fourier
   - cost benefit analysis
   - microeconomics and macroeconomics
   - introduction to machine learning

MCS 472 Lecture 20
Industrial Math & Computation
Jan Verschelde, 27 February 2026

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The Second Project

We are at the end of week seven.

- Presentations happen in week ten.
- Work on the project in weeks eight and nine.
- Commit to a project topic soon.
- Preferably by 5pm on Wednesday 4 March.
- Send email with address of your partner in CC.

proposals for topics

The last five lectures ended with proposals.

1. linear programming
2. regression
3. least squares Chebyshev and Fourier
4. cost-benefit analysis
5. microeconomics and macroeconomics

In addition, topics have been added on the quarternion Fourier transform and machine learning.

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1. the quaternion Fourier transform

The discrete Fourier transform takes on input a sequence of real numbers and returns a sequence of complex numbers.

For input sequences of complex numbers, the Quaternion Fourier Transform (QFT) is defined, see for example the book

Quaternion Fourier Transforms for Signal and Image Processing by T. A. Ell, N. Le Bihan, and S. J. Sangwine, Wiley, 2014.

- Define the mathematical properties of the QFT.
  What are the differences with the Discrete Fourier Transform?
- Develop a basic Julia implementation of the QFT,
  or alternatively, experiment with available code.
- Describe the application of the QFT to color images.

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2. some industrial applications

Read the paper by Ernest Koenigsberg: *Some Industrial Applications of Linear Programming*. Operational Research Society, Vol. 12, No. 2, pages 105–114, 1961.

1. Select one problem and explain the LP formulation, *in your own words.*
2. Work out an example with numerical data.
3. Use JuMP to solve an instance of the same dimensions as in the paper.
4. Report on the computational cost of the problem.

3. managing a production facility

Read the first section of the book by Robert J. Vanderbei: *Linear Programming. Foundations and Extensions*. Fifth Edition, Springer-Verlag, 2020.

1. Explain why the management of a production facility translates into an LP problem, *in your own words.*
2. Setup a program to generate numerical data, for any dimension.
3. Use JuMP to solve the generated instances.
4. Report on the computational cost for various dimensions.

4. solving network flow problems with JuMP

A network is a weighted directed graph allowing for flow to go from a source to a sink along the edges which each have a capacity.

1. Starting with the JuMP tutorial on network flow problems, explore shortest path, assignment, and max-flow problems.
2. Read the paper by
   Shuvomoy Das Gupta, J. Kevin Tobin, and Lacra Pavel on
   A two-step linear programming model for energy-efficient timetables in metro railway networks,
   in *Transportation Research Part B*, vol. 93, pages 57-74, 2016.
3. Explain the making of timetables as a network problem. Elaborate an illustrative example.

5. simplex is not a polynomial-time algorithm

The title of this topic comes from the title of section 8.6 of the book *Combinatorial Optimization. Algorithms and Complexity* by Christos H. Papadimitriou and Kenneth Steiglitz, Dover 1998. Study also **How good is the simplex algorithm?** by Victor Klee and George J. Minty, in *Inequalities III*, pages 159-175, Academic Press, 1972.

1. Formulate the construction of the Klee-Minty cube.
2. Set up the formulation for any dimension to run experiments with the LP solver in GLPK.
3. Do you observe exponential running times for increasing dimensions?

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6. fit processor data

At `https://www.cpubenchmark.net`
we can find data for over a million benchmarked processors.

Consider the following questions:

1. Does Moore's Law still hold?
2. What is the relation between price and performance?

7. fit census data

Using census data from 1790 to today,
find experimentally a best fitting polynomial for the
U.S. population in millions against time in decades.

Consider the following questions:

- What is the best degree of polynomial?
- Instead of one single polynomial,
  would a piecewise polynomial model fit better?

8. fit corona virus data

Use data from the covid-19 pandemic
to model the exponential growth in a surge of infections.

Consider the following questions:

- What is the exponent in the surge?
- Do different surges have different exponents?

9. fit student performance data

Use the Illinois Report Card Data to relate student performance to the expenditures per pupil.

Consider the following questions:

- Does per pupil expenditures predict the graduation rate?
- How does teaching experience relate to student performance?

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10. fit average daily temperatures

Obtain the average temperature for each day of one year.

Fit the data with a trigonometric basis.

Consider the following questions:

- How large should the basis be for an accurate representation?
- Compare the fit of one year on data of another year.
  How good does the fit predict the temperature of the other year?

11. fit USD/EUR currency fluctuations

Consider the evolution of the US dollar versus the Euro.

Which basis would be better? Chebyshev or Fourier?

Consider the following questions:

1. Gather data for at least one decade and fit the data with a Chebyshev and a Fourier basis. Which basis works best?
2. Is there a difference between the long term and short term? Compare one decade versus one month.

12. on the origin of the Chebyshev polynomials

How did Chebyshev arrive at his polynomials?

Consider the paper by V. L. Goncharov:
The Theory of Best Approximation of Functions.
Journal of Approximation Theory 106:2–57, 2000.

The second section of this paper describes the memoir
Théorie des mécanismes connus sous le nom de parallélogrammes,
by Chebyshev, which appeared in 1854.

Consider the following questions:

- Verify the reduction of computing a best approximating polynomial to a differential equation.
- Illustrate your verification with some computational experiments.

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13. Is a doctoral degree financially worthwhile?

Is four years of study beyond the master's to obtain
a doctoral degree financially worthwhile?

Consider the following questions:

- Does the added income throughout the career pay back
  1. the cost of the education, and
  2. the loss of income during the four years of study?
- Is there is a difference between academia and industry?

14. Quality of a bachelors degree.

What is a good university education?

Consider this question from the point of view of quality,
using Taguchi Quality Control.

1. Focus on a particular major and define the expectations in financial terms. What is the expected start salary?
2. Define the parameters in the quality loss function.
3. Gather data and compute the quality loss coefficient.

15. use public transport or your own car?

For the daily commute, compare the cost of using public transport versus using your own car.

- In your study consider the normal life span of a car.
- The cost of public transport includes a fixed fare, subject to annual fare hikes.
- The cost of a car includes not only the purchase price, but also taxes, insurance, fuel, repairs, and depreciation cost.

Consider the following questions:

- What is the total saving of using public transport?
- Explain how the annual increase in saving could be used to justify an annual fare hike, that is then also fair...

16. buy a house or rent?

Suppose you receive a job offer which requires you to move to a different city. Should you buy a house or rent?

- In your study consider a fixed number N of years, which equals the number of years of your job. The N can be used as a parameter.
- Consider the costs and benefits of home ownership.
- Consider the costs and benefits of renting.

Consider the following questions:

- Given the transaction costs of buying a home, what is the minimum value of N to make it worthwhile?
- Home ownership is often cited as a protection against inflation. Consider the scenarios of low and high inflation rate and study the consequences of each in the buying or renting decision.

17. should we colonize Mars?

What are the benefits of manned missions to Mars?

- Make estimates of the financial investments (first costs) required to put humans on the surface of Mars?
- What are the expected benefits to society?

As a point of departure for your study,
gather historical facts and data of the moon landings.
What were the costs and benefits of the moon missions?

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18. Nash equilibria

The concept of a Nash equilibrium is important in game theory.

Read the Game Theory book of Hans Peters for the definition and provide a computational description of a good example.

Consider the following questions.

1. Formulate an economically interesting model.
2. Give an example of an application with many Nash equilibria.

19. model a price war

Develop a computational model of a price war.

Read chapter 11 of *It's a Nonlinear World* by Richard Enns.

Consider the following questions.

1. Formulate a model with a plausible demand function.
2. Is it possible that chaos occurs?

20. interval arithmetic to solve linear systems

Data from real applications often come with noise.

Read the paper **A method for solving systems of linear interval equations applied to the Leontief input-output of economics**, by L. Dymova, P. Sevastjanov, and M. Pilarek, *Expert Systems with Applications*, vol. 40, pages 222–230, 2013.

Consider the following:

1. Summarize the content of the paper, with special attention to the input-output model of economics.
2. Reproduce the computations on the examples in the paper, for example, using SageMath.

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21. machine learning with scikit-learn

Gavin Hackeling: *Mastering Machine Learning with scikit-learn. Apply effective learning algorithms to real-world problems using scikit-learn.* Packt publishing, 2014.

One software that learns from experience is scikit-learn.

1. Read the book and the software documentation.
2. Describe how it fits in the computational ecosystem of Python.
3. Illustrate the capabilities by a good use case.
   How does machine learning predict the housing price?

22. computational geo-cultural modeling

Consider **What Makes Paris Look Like Paris?** by Carl Doersch, Saurabh Singh, Abhinav Gupta, Josef Sivic, and Alexei A. Efros in *Communications of the ACM*, Vol 58, No 12, pages 103-110, 2015.

This authors suggest *computational geo-cultural modeling* as the name for a new research area.

1. Write a summary of the paper.
2. What are the key algorithms needed for the computations?
3. Gather pictures from downtown and a Chicago suburb. What are the distinctive elements in the images?