Notes for Lecture 2 – Wednesday 27 August 2003

introduction to Taguchi Quality Control

These notes are supplemental to §1.6 in the text book, see [1] for more information.

1. Definition of Quality

What it is not: Since quality is used in everyday life, we must make some obvious points:

product quality $\neq$ *product quantity*: This is a cultural issue. The U.S. is a typical throw-away society with a diverse market, whereas Japan is a homogeneous society. The outcome of world war II promoted mass production and consumption in the U.S., while Japan suffered.

quality $\neq$ *value*: Value is subjective and related to the supply-demand-marketing chain. An object (like a worn out teddy bear) can have a tremendous personal value, but no quality.

Definition of Genichi Taguchi (1986):

> "An article of good quality performs its intended functions without variability, and causes little loss through *harmful side effects*, including the cost of using it."

Some observations on this definition:

1. The key point is without variability. Prior to the ideas of Taguchi, people tought the production was okay as long as the products were within the tolerances. The reduction of variability is the prime goal of Taguchi quality control.
2. To enhance the quality we want to reduce loss. But we only care about loss caused by variability in the product, not by loss caused by *harmful side effects*. Consider for example liquor. The quality of a bottle of liquor is its percentage of alcohol, because its intended effect is intoxication. Harmful side effects are the accidents caused by drunk driving or fights, etc...
3. Modeling the loss by a quality loss function is central. We next expand on this.

2. Quality Loss Functions (QLF)

Consider for example the production of shirts, measured by neck size.

Let y be the size of the produced shirt, i.e., its neck size; and m the buyer's exact neck size.

We denote by $L(y)$ the loss due to the difference between y and m. When $y < m$ (the shirt is too small), we have to return the item (or discard it). When $y > m$ (the shirt is too wide), we have to tailor.

Let us now look at the mathematical Taylor expansion of $L(y)$ about m:

$$L(y) = L(m) + \frac{L'(m)}{1!}(y-m) + \frac{L''(m)}{2!}(y-m)^2 + \text{higher order terms.}$$

Now observe the following points:

$L(m) = 0$, because if the neck size is exact, then there is no loss.

$L'(m) = 0$, because the loss is minimal for $y = m$.

So, we may represent the loss by

$$L(y) \approx k(y-m)^2, \quad \text{for some constant } k.$$

The constant k is determined by interpolation, as we illustrate next by a numerical example.

Suppose the critical deviation from m for the shirt being too small occurs at $\Delta m^- = 0.5$cm with corresponding cost of rejection being $L^- = \$40$. Furthermore, the shirt is found too wide when it deviates more than 1 cm from m, i.e., $\Delta m^+ = 1$cm, with an associated cost for tailoring set at $L^+ = \$20$. Since we have different losses when too small or too wide, our quality loss function is piecewise quadratic:

$$L(y) = \begin{cases} k^+(y-m)^2 & y \geq m \\ k^-(y-m)^2 & y < m \end{cases}$$

We determine k^+ as follows: $L^+ = k^+(\Delta m^+)^2 \Rightarrow 20 = k^+(1.0)^2 \Rightarrow k^+ = 20$.
Similarly, k^- is computed as follows: $L^- = k^-(\Delta m^-)^2 \Rightarrow 40 = k^-(0.5)^2 \Rightarrow k^- = 160$.

3. Expected Loss of Quality

Denote by X the random variable modeling the outcome of production. The cumulative probability distribution function is denoted by $F(x)$, with mean $\mu = E[X]$ and standard deviation $\sigma^2 = E[(X-\mu)^2]$. Let θ denote the target value for the production. Then our quality loss function is $L(X,\theta) = k(X-\theta)^2$, for some constant k.

The expected loss is

$$\begin{aligned} E[L(X,\theta)] &= \int_{-\infty}^{+\infty} k(x-\theta)^2 dF(x) \\ &= \int_{-\infty}^{+\infty} k(x-\mu+\mu-\theta)^2 dF(x) \\ &= \int_{-\infty}^{+\infty} k[(x-\mu)+(\mu-\theta)]^2 dF(x) \\ &= \int_{-\infty}^{+\infty} k(x-\mu)^2 dF(x) + \int_{-\infty}^{+\infty} 2k(x-\mu)(\mu-\theta) dF(x) + \int_{-\infty}^{+\infty} k(\mu-\theta)^2 dF(x) \\ &= k\int_{-\infty}^{+\infty} (x-\mu)^2 dF(x) + 2k(\mu-\theta)\left(\int_{-\infty}^{+\infty} x dF(x) - \mu \int_{-\infty}^{+\infty} dF(x)\right) + k(\mu-\theta)^2 \int_{-\infty}^{+\infty} dF(x) \end{aligned}$$

We now use the definitions for $\sigma^2 = \int_{-\infty}^{+\infty} (x-\mu)^2 dF(x)$, $\mu = \int_{-\infty}^{+\infty} x dF(x)$, and $\int_{-\infty}^{+\infty} dF(x) = 1$:

$$\begin{aligned} E[L(X,\theta)] &= k\sigma^2 + 2k(\mu-\theta)(\mu-\mu) + k(\mu-\theta)^2 \\ &= k\sigma^2 + k(\mu-\theta)^2 \end{aligned}$$

The expected loss consists of two terms: the loss due to variability $k\sigma^2$ and $k(\mu-\theta)^2$ is the loss due to missing the target.

References

[1] G. Taguchi. *Introduction to Quality Engineering. Designing Quality into Products and Processes.* Asian Productivity Organization, 1986.