

This is a take home exam. Your answers to the questions below are due on Monday 11 October, at 1PM (start of our regular class meeting). Every question counts for 20 points. Whenever appropriate, the use of Maple is encouraged.

1. Consider the problem of computing $f \bmod \langle p \rangle$, for univariate polynomials p and f .
 - (a) For $p = x^2 - 1$ and $f = x^3 + 1$ compute $f \bmod \langle p \rangle$, using two different algorithms.
 - (b) For which instances of p do the two algorithms lead to ill-conditioned problems?
2. Consider the system $P(x, y) = \begin{cases} x^3 + y^2 - xy = 0 \\ x - y + 1 = 0. \end{cases}$
 - (a) For the basis $\mathbf{b} = (1, x, x^2)$, compute A_x . Although you may verify your result with Maple, show how – for this specific system – you can compute A_x efficiently.
 - (b) What are the eigenvalues of A_x ? Give a short argument to explain the relationship between the eigenvalues of A_x and the zeros of the system.
 - (c) Suppose one could vary the coefficients of the second equation of P . Formulate a geometric condition on the second equation for which the eigenvalue problem for A_x becomes ill conditioned. Illustrate with a plot.
3. Consider the polynomial $p = x^3 + ax^2 + b$, with parameters a and b .
 - (a) Derive a condition on the parameters a and b for which p has singular solutions.
 - (b) For which values of a and b are all three roots of p real?
4. Consider $p = x^3 - x^2 - 1.000001x + .999999$.

Find the smallest possible change to the coefficients of p so that 1 becomes a double root. Use standard Euclidean norm to measure the distance between the coefficients. The leading coefficient of p must remain one.
5. Consider the polynomials $f = x^4 + 0.001x^3 - 2.003002x^2 - 0.001x + 1.003002$ and $g = x^4 - 0.001x^3 - 2.003002x^2 + 0.001x + 1.003002$.
 - (a) Determine the degree of $\text{GCD}(f, g)$, using Sylvester subresultant matrices.
 - (b) What can you say about the conditioning of this problem, for these two polynomials f and g ?