Review of Chapters 1 to 6

The midterm exam is a take home exam. The questions of the exam will be handed out on Friday 8 October at 1PM in class. Your answers will be due the following Monday, October 11, at 1PM in class.

Below are some questions to review the materials of the first six chapters of the textbook. The lecture notes are as important as the textbook.

1. (a) Describe two different ways to compute f mod $\langle p \rangle$.
   Illustrate with $f = x^3 - 2$ and $p = x^2 - 3x + 2$.
   (b) In the multivariate case (replace p by P), which way seems easiest to you? Why?
   In your arguments refer to the numerical stability of the algorithms on empirical polynomials.
2. (a) Formulate a specific version of the "central theorem" in the case of a zero dimensional ideal where all solutions are regular, i.e.: all multiplicities are equal to one.
   (b) Illustrate this specific theorem by taking a system of two quadratic polynomials. Choose each polynomial as a product of two general linear equations.
3. Show that, if $\bar{z}$ is an approximate zero of an empirical univariate polynomial $\bar{p}$, close to a root of multiplicity m, we must know the coefficients of $\bar{p}$ with an accuracy of at least $k \times m$ decimal places in order to have k decimal places accurate in the approximate zero $\bar{z}$.
4. State the Chebyshev criterium. What is its relevance to the rootfinding problem?
5. Consider the following polynomial: $p(x) = x^5 - 2.1249x^4 + 1.7872x^3 + 0.15318x^2 - 0.74380x - 0.012491$.
   As we can see, the coefficients of p are given with five correct decimal places.
   The solutions computed by Maple (in its default working precision of ten decimal places) are

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0.3271873770 - 0.01518894248 I, 0.3271873770 + 0.01518894248 I,
0.4517306006 - 0.03552775934 I, 0.4517306006 + 0.03552775934 I, 0.5670640449
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   How many decimal places in the solutions can you trust?
   Is it possible that all solutions are real? Compute a backward error.
6. Discuss the conditioning of polynomial division.
   For which polynomials is the division ill-conditioned?
7. Give an expression to find the discriminant of a general quintic $p(x) = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$.
8. Set up and solve the linear system to determine the coefficients of A and B satisfying $Af + Bg = 0$, for $f = x^4 + 2x^3 - 9x^2 - 2x + 8$ and $g = x^4 + 2x^3 - 7x^2 - 8x + 12$.
   Verify your results computing GCD(f, g).
9. What is a pejorative condition number? Define the problem so your answer is self-contained.
10. For which input polynomials is the approximate GCD ill-conditioned?