The goal of the course is to study symbolic-numerical algorithms with their implementation and applications to science and engineering. Computational algebraic geometry has many emerging applications and offers a fun way to learn more about algebraic geometry. A previous run of the course used “Numerical Polynomial Algebra” by H.J. Stetter as textbook. While we will study many of the topics covered by this book, there will be lecture notes, covering the following topics listed below:

Numerical algorithms:

+ linear algebra: singular value decomposition and rank revealing algorithms;
+ path following algorithms for nonlinear problems to obtain globally convergent solvers.

Symbolic-numeric algorithms:

+ greatest common divisor (GCD) for approximate inputs;
+ application of this approximate GCD to (1) finding multiple roots of univariate polynomials, (2) a numerical Jordan canonical form;
+ approximate absolute factorization of multivariate polynomials;
+ computation of the multiplicity structure of a root of a polynomial system.

An introduction to computational algebraic geometry:

+ elimination methods and Groebner bases;
+ homotopy methods following the theorems of Bézout and Bernshtein;
+ polyhedral methods and “tropical geometry”.

Other topics:

+ algoritmic differentiation;
+ conditioning, complexity, and stability.

Applications:

+ polynomial systems arising in robotic mechanical design, game theory, control, geometric modelling, etc.

Bibliography:

(1) J. Angeles: *Fundamentals of Robotic Mechanical Systems: Theory, Methods, and Algorithms.* 2nd edition, Springer, 2003.
(2) L. Blum, F. Cucker, M. Shub, and S.Smale: *Complexity and Real Computation.* Springer, 1997.
(3) D. Cox, J. Little, and D. O’Shea: *Ideals, Varieties, and Algorithms. An introduction to Computational Algebraic Geometry and Commutative Algebra.* Second Edition, Springer, 1997.
(4) J.W. Demmel: *Applied Linear Algebra.* SIAM, 1997.
(5) G.-M. Greuel and G. Pfister: *A Singular Introduction to Commutative Algebra.* Springer, 2002.
(6) A. Griewank: *Evaluating derivatives: principles and techniques of algorithmic differentiation.* SIAM, 2000.
(7) A.J. Sommese and C.W. Wampler: *The Numerical solution of systems of polynomials arising in engineering and science.* World Scientific, 2005.
(8) H.J. Stetter: *Numerical Polynomial Algebra.* SIAM, 2004.
(9) B. Sturmfels: *Solving Systems of Polynomial Equations.* AMS, 2002.