

Review 2

1 Overview of part II of the Course

- singularities, algebraic sets, tropical geometry ...

2 Questions of Part I

- a Gröbner basis
- the Newton-Puiseux method
- counting the number of solutions
- quadratic turning points

3 Questions of Part II

- the pejorative manifold
- multiplicity of an isolated solution
- witness set
- the linear trace
- primary decomposition
- the tropical prevariety

MCS 563 Lecture 42
Analytic Symbolic Computation
Jan Verschelde, 28 April 2014

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overview of the second part of the course

The focus on the first part of the course was on

- symbolic, numeric, and polyhedral methods
- to compute all isolated solutions of a polynomial system.

In the second part, we turned our attention to

- singular solutions of polynomial systems;
- positive dimensional solution sets:
 - ▶ factorization methods;
 - ▶ numerical irreducible decomposition;
 - ▶ primary decomposition.
- topics like tropical algebraic geometry.

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a Gröbner basis

Consider an ideal I in a polynomial ring $\mathbb{Q}[x_1, x_2, \dots, x_n]$.

Give a concise but precise definition of a Gröbner basis G for I .

What are the defining characteristics of G ? Explain one application.

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the Newton-Puiseux method

Consider $f = x^3 + 2x^4y - xy^3 + y^6 \in \mathbb{C}[[x]][y]$.

Draw the Newton polyhedron of f and find the leading powers for the Puiseux series expansions.

Describe the computation of the first and second term of a Puiseux series, using f to illustrate your description.

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counting the number of solutions

Consider the system

$$\begin{cases} x_1^2 x_2^3 - 1 = 0 \\ x_1^3 x_2^5 - 1 = 0 \end{cases}$$

How many solutions does this system have? Justify your count.

Explain how to solve this system.

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quadratic turning points

What is a quadratic turning point?

Illustrate the definition with an appropriate example.

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pejorative manifold

What is the pejorative condition number? Give a definition.

Illustrate the definition with an appropriate example.

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multiplicity of an isolated solution

Consider an isolated solution of a polynomial and give a definition of its multiplicity.

Your definition should be general enough so it applies not just to square but also to overdetermined systems.

Illustrate your definition with an example.

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witness set

What is a witness set? Give a definition.

In what context do witness sets occur?

Illustrate the application of witness sets with an example.

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the linear trace

What is the linear trace? Give a definition.

Give an application where the linear trace is used.

In particular, what problem can we solve with the linear trace?

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primary decomposition

What is an associated prime? Give a definition.

Illustrate with an example the role of associate primes in a primary decomposition.

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the tropical prevariety

What is the tropical prevariety? Give a definition.

Illustrate with an example the distinction between the tropical prevariety and the tropical variety.