Barriers for Synchronizations

1. Synchronizing Computations
   - the linear barrier
   - tree and butterfly barriers
   - the `sendrecv` method of MPI

2. the Prefix Sum Algorithm
   - data parallel computations
   - the prefix sum algorithm in MPI

3. Brent’s Theorem
   - parallel random access machine model
   - application to parallel summation

MCS 572 Lecture 28
Introduction to Supercomputing
Jan Verschelde, 15 March 2023
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synchronization barrier

In synchronized computations, processors pass through a number of stages in an algorithm.

**Definition**

A *synchronization barrier* guarantees that no processor continues to the next stage until all processors have finished the current stage.

In the definition, the “processor” stands for a process, thread, or task.

**Examples:**

- Message passing defines `MPI_Barrier(MPI_Comm comm)`.
- OpenMP has the `#pragma omp barrier` construct.
- CUDA provides the instruction `__syncthreads()`.
the linear barrier

A barrier has two phases:
1. the arrival or trapping phase; and
2. the departure or release phase.

The manager maintains a counter: only when all workers have sent to the manager, does the manager send messages to all workers.

```
for i from 1 to p − 1 do
    receive from i
```

```
for i from 1 to p − 1 do
    send to i
```

send to manager
receive from manager

The counter implementation of a barrier or linear barrier is effective but it takes $O(p)$ steps.
the linear barrier for $p = 8$
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the tree barrier for $p = 8$
implementing a tree barrier

The trapping phase, for $p = 2^k$ (recall the fan in gather):

for $i$ from $k - 1$ down to 0 do
  for $j$ from $2^i$ to $2^{i+1}$ do
    node $j$ sends to node $j - 2^i$
    node $j - 2^i$ receives from node $j$

The release phase, for $p = 2^k$ (recall the fan out scatter):

for $i$ from 0 to $k - 1$ do
  for $j$ from 0 to $2^i - 1$ do
    node $j$ sends to $j + 2^i$
    node $j + 2^i$ receives from node $j$

The tree barrier needs $2 \log_2(p)$ stages.

Number of messages: $2 \sum_{i=0}^{k-1} 2^i = 2 \left( \frac{2^k - 1}{2 - 1} \right) = 2^{k+1} - 2 = 2p - 2$. 
the butterfly barrier for $p = 8$

Two processors can synchronize in one step:

Applied to $p = 4$ and $p = 8$, observe there are no idle processors:
the algorithm for a butterfly barrier, for $p = 2^k$

for $i$ from 0 to $k - 1$ do
    $s := 0$
    for $j$ from 0 to $p - 1$ do
        if $(j \mod 2^{i+1} = 0)$ $s := j$
        node $j$ sends to node $((j + 2^i) \mod 2^{i+1}) + s$
        node $((j + 2^i) \mod 2^{i+1}) + s$ receives from node $j$

\begin{center}
\begin{tikzpicture}
\foreach \i in {0,1,2,3,4,5,6,7} {
    \node (P\i) at (\i,7) {$P_{\i}$};
    \foreach \j in {0,1,2,3,4,5,6,7} {
        \ifnum\i<\j
            \draw[->] (P\i) -- (P\j);
        \fi
    }
}
\node (time) at (3,-2) {time};
\end{tikzpicture}
\end{center}
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avoiding deadlock with `sendrecv`

\[
\begin{array}{c}
\begin{align*}
P_{i-1} & & P_i & & P_{i+1} \\
\text{recv}(P_i) & \longrightarrow & \text{send}(P_{i-1}) & & \\
\text{send}(P_i) & \longrightarrow & \text{recv}(P_{i-1}) & \longrightarrow & \text{recv}(P_{i+1}) & \longrightarrow & \text{send}(P_i)
\end{align*}
\end{array}
\]

is equivalent to

\[
\begin{array}{c}
\begin{align*}
P_{i-1} & & P_i & & P_{i+1} \\
\text{sendrecv}(P_i) & \longleftrightarrow & \text{sendrecv}(P_{i-1}) & & \\
& & \text{sendrecv}(P_{i+1}) & \longleftrightarrow & \text{sendrecv}(P_i)
\end{align*}
\end{array}
\]
the `sendrecv` in MPI

```c
MPI_Sendrecv(sendbuf, sendcount, sendtype, dest, sendtag,
             recvbuf, recvcount, recvtype, source, recvtag,
             comm, status)
```

where the parameters are

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>sendbuf</code></td>
<td>initial address of send buffer</td>
</tr>
<tr>
<td><code>sendcount</code></td>
<td>number of elements in send buffer</td>
</tr>
<tr>
<td><code>sendtype</code></td>
<td>type of elements in send buffer</td>
</tr>
<tr>
<td><code>dest</code></td>
<td>rank of destination</td>
</tr>
<tr>
<td><code>sendtag</code></td>
<td>send tag</td>
</tr>
<tr>
<td><code>recvbuf</code></td>
<td>initial address of receive buffer</td>
</tr>
<tr>
<td><code>recvcount</code></td>
<td>number of elements in receive buffer</td>
</tr>
<tr>
<td><code>recvtype</code></td>
<td>type of elements in receive buffer</td>
</tr>
<tr>
<td><code>source</code></td>
<td>rank of source or <code>MPI_ANY_SOURCE</code></td>
</tr>
<tr>
<td><code>recvtag</code></td>
<td>receive tag or <code>MPI_ANY_TAG</code></td>
</tr>
<tr>
<td><code>comm</code></td>
<td>communicator</td>
</tr>
<tr>
<td><code>status</code></td>
<td>status object</td>
</tr>
</tbody>
</table>
We use MPI_Sendrecv to synchronize two nodes:

```
$ mpirun -np 2 ./use_sendrecv
Node 0 will send a to 1
Node 0 received b from 1
Node 1 will send b to 0
Node 1 received a from 0
$ 
```
using MPI_Sendrecv

#include <stdio.h>
#include <mpi.h>
#define sendtag 100

int main ( int argc, char *argv[]) {
  int i,j;
  MPI_Status status;

  MPI_Init(&argc,&argv);
  MPI_Comm_rank(MPI_COMM_WORLD,&i);

  j = (i+1) % 2; /* the other node */
a bidirectional data transfer

Processors 0 and 1 swap characters:

```c
{
    char c = 'a' + (char)i; /* send buffer */
    printf("Node %d will send %c to %d\n", i, c, j);
    char d; /* receive buffer */

    MPI_Sendrecv(&c,1,MPI_CHAR,j,sendtag,
                 &d,1,MPI_CHAR,MPI_ANY_SOURCE,
                 MPI_ANY_TAG,MPI_COMM_WORLD,&status);

    printf("Node %d received %c from %d\n", i, d, j);
}

MPI_Finalize();
return 0;
```
Sendrecv is one of the methods of mpi4py.MPI.Comm.

Another method is sendrecv (lowercase) which is more generic, i.e.: works for any Python object instead of with numpy arrays.

MPI.jl is still under development ...

Asking for help on MPI.sendrecv returns

Binding MPI.sendrecv does not exist.
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data parallel computations

A data parallel computation is a computation where the same operations are performed on different data simultaneously.

Benefits:

- easy to program,
- scales well,
- fit for SIMD computers.

Problem: compute $\sum_{i=0}^{n-1} a_i$ for $n = p = 2^k$.

Related problem: composite trapezoidal rule.
the prefix sum for $n = p = 8$

\[
\begin{array}{cccccccc}
  & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\
\hline
\text{step 1} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\sum_{i=0}^0 & \sum_{i=0}^1 & \sum_{i=1}^2 & \sum_{i=2}^3 & \sum_{i=3}^4 & \sum_{i=4}^5 & \sum_{i=5}^6 & \sum_{i=6}^7 \\
\text{step 2} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\sum_{i=0}^0 & \sum_{i=0}^1 & \sum_{i=0}^2 & \sum_{i=1}^3 & \sum_{i=2}^4 & \sum_{i=3}^5 & \sum_{i=4}^6 & \sum_{i=4}^7 \\
\text{step 3} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\sum_{i=0}^0 & \sum_{i=0}^1 & \sum_{i=0}^2 & \sum_{i=0}^3 & \sum_{i=0}^4 & \sum_{i=0}^5 & \sum_{i=0}^6 & \sum_{i=0}^7 \\
\end{array}
\]
the prefix sum algorithm

For \( n = p = 2^k \), processor \( i \) executes:

\[
\begin{align*}
  s &:= 1; \quad x := a_i; \\
  \text{for } j \text{ from } 0 \text{ to } k - 1 \text{ do} \\
  &\text{if } (j < p - s + 1) \text{ send } x \text{ to processor } i + s; \\
  &\text{if } (j > s - 1) \text{ receive } y \text{ from processor } i - s; \\
  &\quad \text{add } y \text{ to } x: \quad x := x + y; \\
  s &:= 2 \times s.
\end{align*}
\]

The speedup: \( \frac{p}{\log_2(p)} \).

Communication overhead: one send/recv in every step.
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#include <stdio.h>
#include "mpi.h"
#define tag 100    /* tag for send/recv */

int main ( int argc, char *argv[] )
{
    int i,j,nb,b,s;
    MPI_Status status;
    const int p = 8;   /* run for 8 processors */

    MPI_Init(&argc,&argv);
    MPI_Comm_rank(MPI_COMM_WORLD,&i);

    nb = i+1;         /* node i holds number i+1 */
    s = 1;            /* shift s will double in every step */
the prefix sum loop

for(j=0; j<3; j++) /* 3 stages, as log2(8) = 3 */ {
    if(i < p - s) /* every one sends, except last s ones */
        MPI_Send(&nb,1,MPI_INT,i+s,tag,MPI_COMM_WORLD);
    if(i >= s) /* every one receives, except first s ones */
    {
        MPI_Recv(&b,1,MPI_INT,i-s,tag,MPI_COMM_WORLD,&status);
        nb += b; /* add received value to current number */
    }
    MPI_Barrier(MPI_COMM_WORLD); /* synchronize computations */
    if(i < s)
        printf("At step %d, node %d has number %d.\n",j+1,i,nb);
    else
        printf("At step %d, Node %d has number %d = %d + %d.\n",j+1,i,nb,nb-b,b);
    s *= 2; /* double the shift */
}
if(i == p-1) printf("The total sum is %d.\n",nb);

running the code

$ mpirun -np 8 ./prefix_sum
At step 1, node 0 has number 1.
At step 1, Node 1 has number 3 = 2 + 1.
At step 1, Node 2 has number 5 = 3 + 2.
At step 1, Node 3 has number 7 = 4 + 3.
At step 1, Node 7 has number 15 = 8 + 7.
At step 1, Node 4 has number 9 = 5 + 4.
At step 1, Node 5 has number 11 = 6 + 5.
At step 1, Node 6 has number 13 = 7 + 6.
At step 2, node 0 has number 1.
At step 2, node 1 has number 3.
At step 2, Node 2 has number 6 = 5 + 1.
At step 2, Node 3 has number 10 = 7 + 3.
At step 2, Node 4 has number 14 = 9 + 5.
At step 2, Node 5 has number 18 = 11 + 7.
At step 2, Node 6 has number 22 = 13 + 9.
At step 2, Node 7 has number 26 = 15 + 11.
At step 3, node 0 has number 1.
At step 3, node 1 has number 3.
At step 3, node 2 has number 6.
At step 3, node 3 has number 10.
At step 3, Node 4 has number 15 = 14 + 1.
At step 3, Node 5 has number 21 = 18 + 3.
At step 3, Node 6 has number 28 = 22 + 6.
At step 3, Node 7 has number 36 = 26 + 10.
The total sum is 36.
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PRAM

PRAM = Parallel Random Access Machine

The PRAM model is an idealized construct.

- It assumes any number of processors can access any items in memory instantly.
- An operation takes one unit time.

The PRAM model helps to derive bounds on the theoretical time of a parallel algorithm.
Brent’s theorem

Assume

1. a parallel computer where each processor can perform an arithmetic operation in unit time; and
2. the computer has exactly enough processors to exploit the maximum concurrency in an algorithm with \( N \) operations, such that \( T \) time steps suffice,

then a computer with \( P \) processors can perform the algorithm in time

\[
T_P \leq T + \frac{N - T}{P},
\]

where \( P \) is less than or equal to the number of processors needed to exploit the maximum concurrency in the algorithm.
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application to parallel summation

Consider the sum of $n$ numbers.

If $n = 2^T$, then the PRAM can do the sum in $T$ steps.

If the PRAM has $P$ processors and $P \leq n/2$, then

$$T_P \leq \lceil \log_2(n) \rceil + \frac{(n - 1) - \log_2(n)}{P},$$

where $T_P$ is the execution time with $P$ processors.

Typically, the number of processors is fixed, and then we want to find the best size $n$ of the problem so the theoretical bounds on the execution time are within reach.


We started chapter 6 in the book of Wilkinson and Allen.

Exercises:

1. Write code using `MPI_sendrecv` in C or Python for a butterfly barrier. Show that your code works for $p = 8$.

2. Rewrite `prefix_sum.c` using `MPI_sendrecv`.

3. Consider the composite trapezoidal rule for the approximation of $\pi$, doubling the number of intervals in each step. Can you apply the prefix sum algorithm so that at the end, processor $i$ holds the approximation for $\pi$ with $2^i$ intervals?