Barriers for Synchronizations

1. Synchronizing Computations
   - the linear barrier
   - tree and butterfly barriers
   - the `sendrecv` method of MPI
2. the Prefix Sum Algorithm
   - data parallel computations
   - the prefix sum algorithm in MPI
3. Brent's Theorem
   - parallel random access machine model
   - application to parallel summation

MCS 572 Lecture 28
Introduction to Supercomputing
Jan Verschelde, 15 March 2023

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synchronization barrier

In synchronized computations, processors pass through a number of stages in an algorithm.

Definition

A *synchronization barrier* guarantees that no processor continues to the next stage until all processors have finished the current stage.

In the definition, the "processor" stands for a process, thread, or task.

Examples:

- Message passing defines `MPI_Barrier(MPI_Comm comm)`.
- OpenMP has the `#pragma omp barrier` construct.
- CUDA provides the instruction `__syncthreads()`.

the linear barrier

A barrier has two phases:

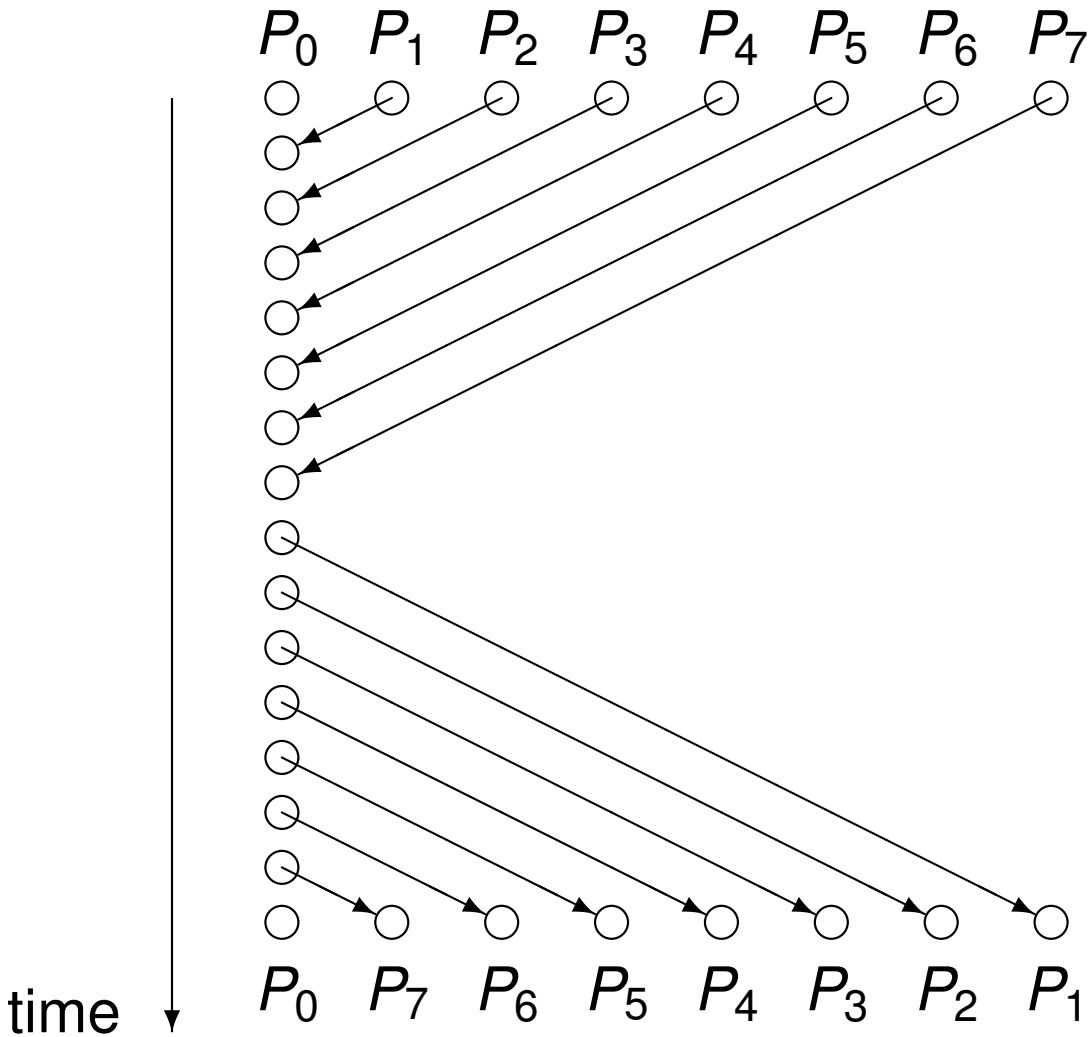
1. the arrival or trapping phase; and
2. the departure or release phase.

The manager maintains a counter: only when all workers have sent to the manager, does the manager send messages to all workers.

manager	worker
for i from 1 to $p-1$ do	
receive from i	send to manager
for i from 1 to $p-1$ do	
send to i	receive from manager

The counter implementation of a barrier or linear barrier is effective but it takes $O(p)$ steps.

the linear barrier for $p = 8$



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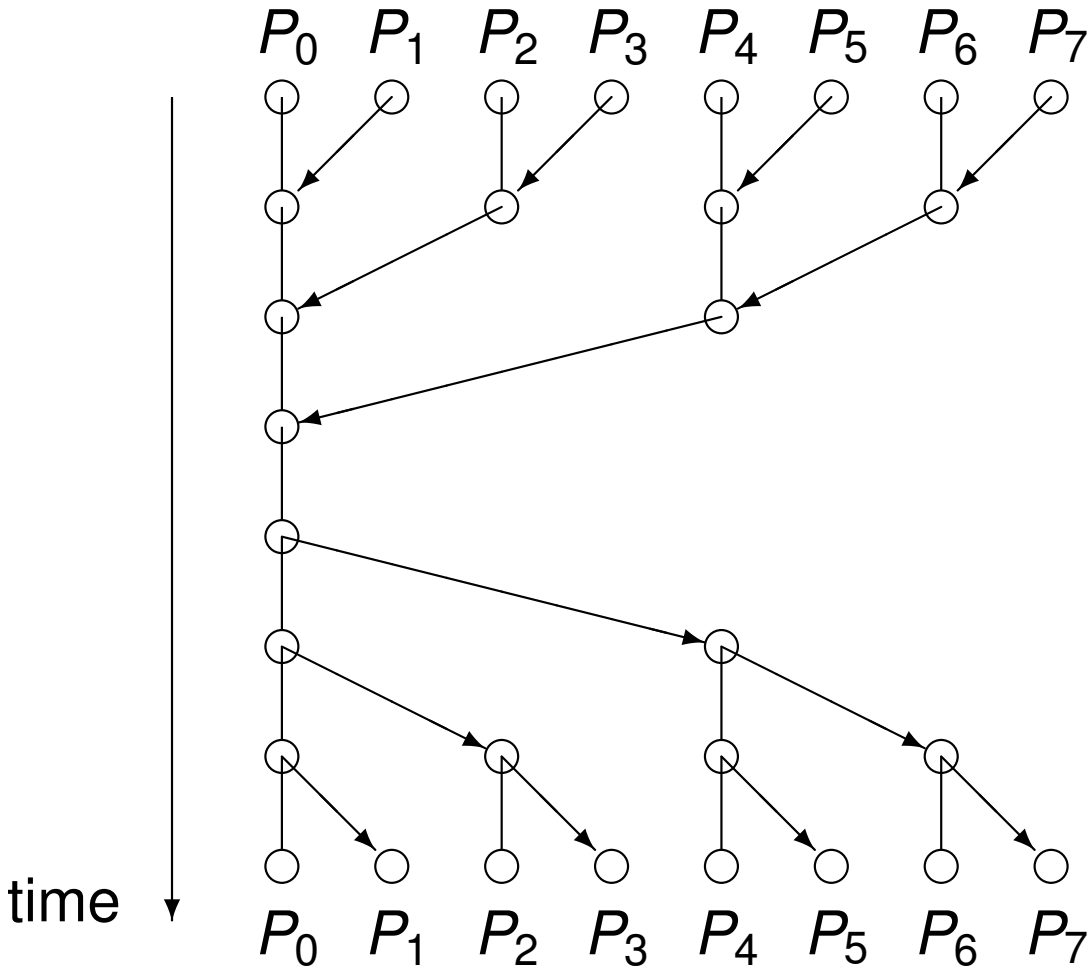
the tree barrier for $p = 8$



implementing a tree barrier

The trapping phase, for $p = 2^k$ (recall the fan in gather):

```
for i from k − 1 down to 0 do
    for j from 2^i to 2^(i+1) do
        node j sends to node j − 2^i
        node j − 2^i receives from node j
```

The release phase, for $p = 2^k$ (recall the fan out scatter):

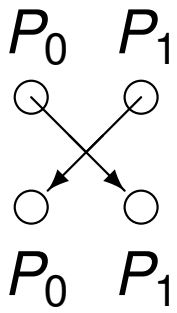
```
for i from 0 to k − 1 do
    for j from 0 to 2^i − 1 do
        node j sends to j + 2^i
        node j + 2^i receives from node j
```

The tree barrier needs $2\log_2(p)$ stages.

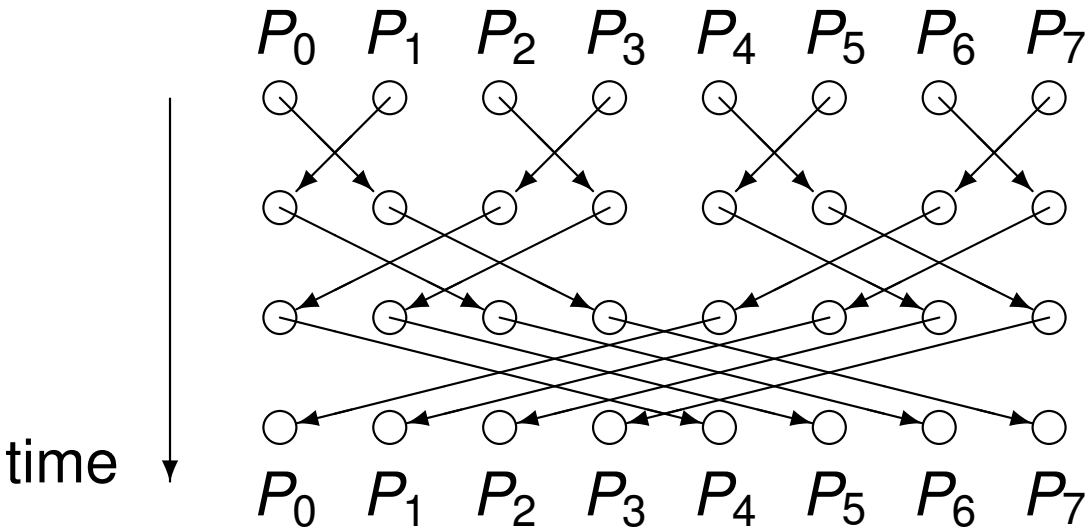
Number of messages: $2\sum_{i=0}^{k-1} 2^i = 2\left(\frac{2^k - 1}{2 - 1}\right) = 2^{k+1} - 2 = 2p - 2.$

the butterfly barrier for $p = 8$

Two processors can synchronize in one step:

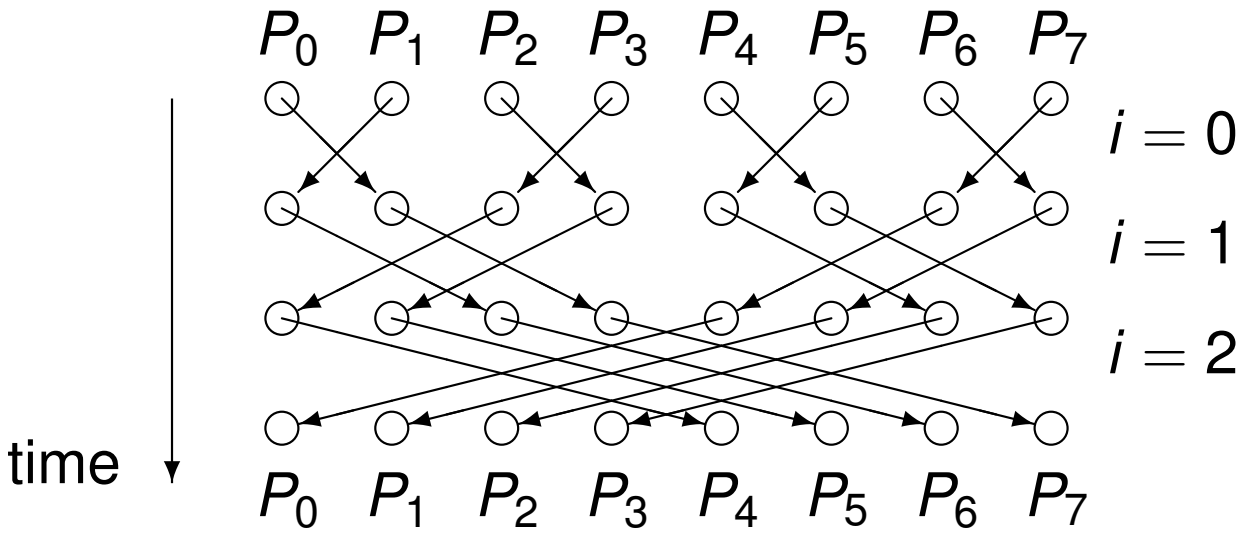


Applied to $p = 4$ and $p = 8$, observe there are no idle processors:



the algorithm for a butterfly barrier, for $p = 2^k$

for i from 0 to $k - 1$ do
  $s := 0$
  for j from 0 to $p - 1$ do
    if ($j \bmod 2^{i+1} = 0$) $s := j$
    node j sends to node $((j + 2^i) \bmod 2^{i+1}) + s$
    node $((j + 2^i) \bmod 2^{i+1}) + s$ receives from node j



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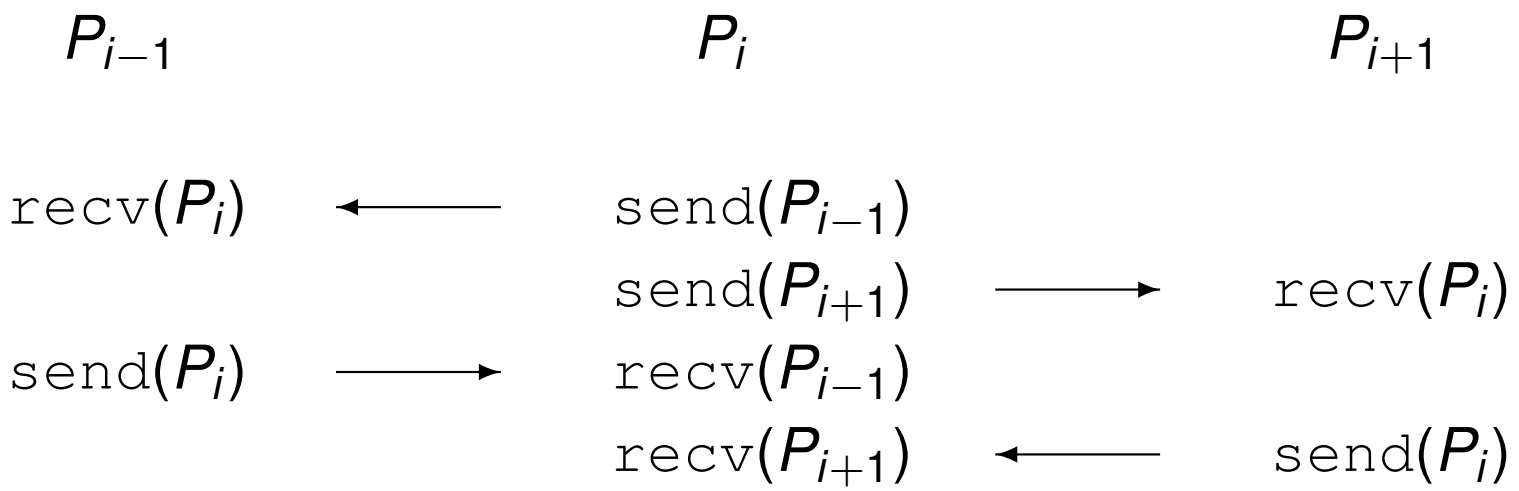
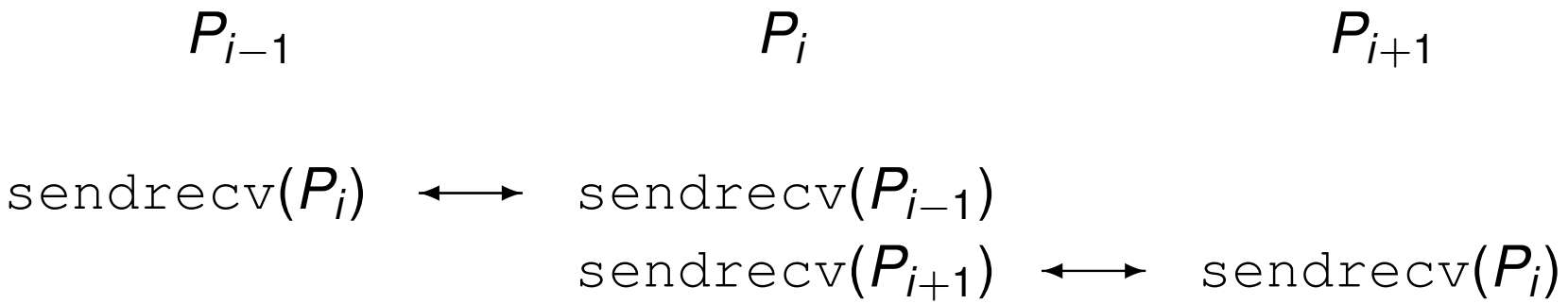
avoiding deadlock with `sendrecv`



is equivalent to



the `sendrecv` in MPI

```
MPI_Sendrecv(sendbuf,sendcount,sendtype,dest,sendtag,
             recvbuf,recvcount,recvtype,source,recvtag,
             comm,status)
```

where the parameters are

`sendbuf`	initial address of send buffer
`sendcount`	number of elements in send buffer
`sendtype`	type of elements in send buffer
`dest`	rank of destination
`sendtag`	send tag
`recvbuf`	initial address of receive buffer
`recvcount`	number of elements in receive buffer
`sendtype`	type of elements in receive buffer
`source`	rank of source or `MPI_ANY_SOURCE`
`recvtag`	receive tag or `MPI_ANY_TAG`
`comm`	communicator
`status`	status object

a simple illustration

We use `MPI_Sendrecv` to synchronize two nodes:

```
$ mpirun -np 2 ./use_sendrecv
Node 0 will send a to 1
Node 0 received b from 1
Node 1 will send b to 0
Node 1 received a from 0
$
```

using MPI_Sendrecv

```
#include <stdio.h>
#include <mpi.h>

#define sendtag 100

int main ( int argc, char *argv[] )
{
   int i,j;
   MPI_Status status;

   MPI_Init(&argc,&argv);
   MPI_Comm_rank(MPI_COMM_WORLD,&i);

   j = (i+1) % 2;  /* the other node */
```

a bidirectional data transfer

Processors 0 and 1 swap characters:

```
{
   char c = 'a' + (char)i; /* send buffer */
   printf("Node %d will send %c to %d\n",i,c,j);
   char d;                 /* receive buffer */

   MPI_Sendrecv(&c,1,MPI_CHAR,j,sendtag,
                &d,1,MPI_CHAR,MPI_ANY_SOURCE,
                MPI_ANY_TAG,MPI_COMM_WORLD,&status);

   printf("Node %d received %c from %d\n",i,d,j);
}

MPI_Finalize();
return 0;
```

sendrecv in mpi4py and MPI.jl

- `Sendrecv` is one of the methods of `mpi4py.MPI.Comm`.

  Another method is `sendrecv` (lowercase) which is more generic, i.e.: works for any Python object instead of with numpy arrays.

- `MPI.jl` is still under development ...

  Asking for help on `MPI.sendrecv` returns

  > Binding `MPI.sendrecv` does not exist.

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data parallel computations

A data parallel computation is a computation where the ***same*** operations are preformed on ***different*** data ***simultaneously***.

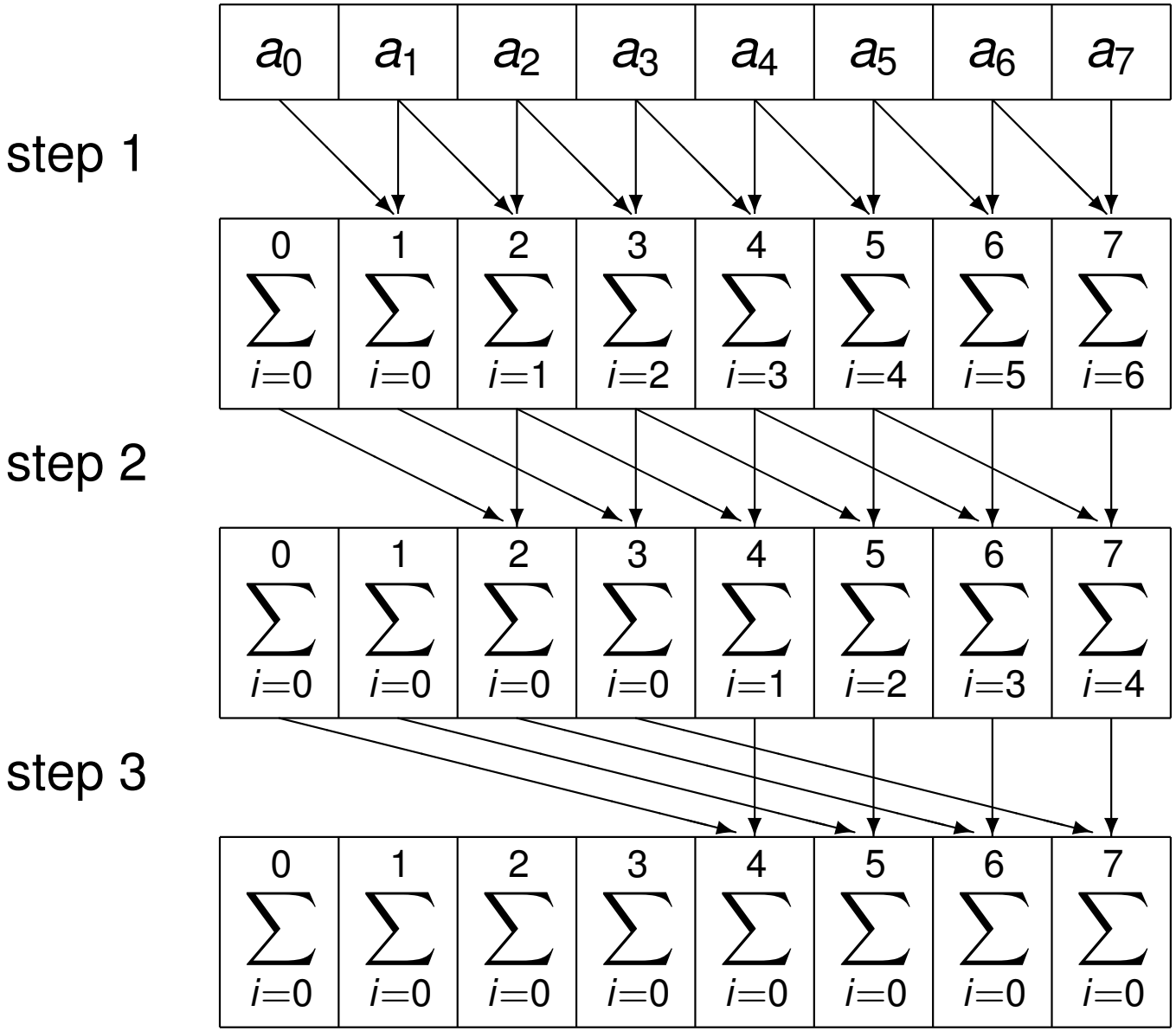
Benefits:

- easy to program,
- scales well,
- fit for SIMD computers.

Problem: compute $\sum_{i=0}^{n-1} a_i$ for $n = p = 2^k$.

Related problem: composite trapezoidal rule.

the prefix sum for $n = p = 8$



the prefix sum algorithm

For $n = p = 2^k$, processor i executes:

$s := 1$; $x := a_i$;
for j from 0 to $k - 1$ do
    if ($j < p - s + 1$) send x to processor $i + s$;
    if ($j > s - 1$) receive y from processor $i - s$;
                add y to x: $x := x + y$;
    $s := 2 \star s$.

The speedup: $\frac{p}{\log_2(p)}$.

Communication overhead: one send/recv in every step.

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MPI code

```
#include <stdio.h>
#include "mpi.h"
#define tag 100                    /* tag for send/recv */

int main ( int argc, char *argv[] )
{
   int i,j,nb,b,s;
   MPI_Status status;
   const int p = 8;          /* run for 8 processors */

   MPI_Init(&argc,&argv);
   MPI_Comm_rank(MPI_COMM_WORLD,&i);

   nb = i+1;                 /* node i holds number i+1 */
   s = 1;      /* shift s will double in every step */
```

the prefix sum loop

```
for(j=0; j<3; j++)                    /* 3 stages, as log2(8) = 3 */
{
   if(i < p - s)      /* every one sends, except last s ones */
      MPI_Send(&nb,1,MPI_INT,i+s,tag,MPI_COMM_WORLD);
   if(i >= s)      /* every one receives, except first s ones */
   {
      MPI_Recv(&b,1,MPI_INT,i-s,tag,MPI_COMM_WORLD,&status);
      nb += b;          /* add received value to current number */
   }
   MPI_Barrier(MPI_COMM_WORLD);  /* synchronize computations */
   if(i < s)
      printf("At step %d, node %d has number %d.\n",j+1,i,nb);
   else
      printf("At step %d, Node %d has number %d = %d + %d.\n",
             j+1,i,nb,nb-b,b);
   s *= 2;                                  /* double the shift */
}
if(i == p-1) printf("The total sum is %d.\n",nb);
```

running the code

```
$ mpirun -np 8 ./prefix_sum
At step 1, node 0 has number 1.
At step 1, Node 1 has number 3 = 2 + 1.
At step 1, Node 2 has number 5 = 3 + 2.
At step 1, Node 3 has number 7 = 4 + 3.
At step 1, Node 7 has number 15 = 8 + 7.
At step 1, Node 4 has number 9 = 5 + 4.
At step 1, Node 5 has number 11 = 6 + 5.
At step 1, Node 6 has number 13 = 7 + 6.
At step 2, node 0 has number 1.
At step 2, node 1 has number 3.
At step 2, Node 2 has number 6 = 5 + 1.
At step 2, Node 3 has number 10 = 7 + 3.
At step 2, Node 4 has number 14 = 9 + 5.
At step 2, Node 5 has number 18 = 11 + 7.
At step 2, Node 6 has number 22 = 13 + 9.
At step 2, Node 7 has number 26 = 15 + 11.
```

running the code continued

```
At step 3, node 0 has number 1.
At step 3, node 1 has number 3.
At step 3, node 2 has number 6.
At step 3, node 3 has number 10.
At step 3, Node 4 has number 15 = 14 + 1.
At step 3, Node 5 has number 21 = 18 + 3.
At step 3, Node 6 has number 28 = 22 + 6.
At step 3, Node 7 has number 36 = 26 + 10.
The total sum is 36.
```

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PRAM

PRAM = Parallel Random Access Machine

The PRAM model is an idealized construct.

- It assumes any number of processors can access any items in memory instantly.
- An operation takes one unit time.

The PRAM model helps to derive bounds on the theoretical time of a parallel algorithm.

Brent's theorem

Assume

1. a parallel computer where each processor can perform an arithmetic operation in unit time; and
2. the computer has exactly enough processors to exploit the maximum concurrency in an algorithm with N operations, such that T time steps suffice,

then a computer with P processors can perform the algorithm in time

$$T_P \leq T + \frac{N - T}{P},$$

where P is less than or equal to the number of processors needed to exploit the maximum concurrency in the algorithm.

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application to parallel summation

Consider the sum of n numbers.

If $n = 2^T$, then the PRAM can do the sum in T steps.

If the PRAM has P processors and $P \leq n/2$, then

$$T_P \leq \lceil \log_2(n) \rceil + \frac{(n-1) - \log_2(n)}{P},$$

where T_P is the execution time with P processors.

Typically, the number of processors is fixed,
and then we want to find the best size n of the problem
so the theoretical bounds on the execution time are within reach.

references

- R. P. Brent: **The parallel evaluation of general arithmetic expressions**. *Journal of the ACM* 12(2): 201-206, 1974.
- John Gustafson: **Brent's Theorem**.
  In *Enclopedia of Parallel Computing*, edited by David Padua, pages 182–185, Springer 2011.
- W. Daniel Hillis and Guy L. Steele: **Data Parallel Algorithms.** *Communications of the ACM*, 29(12): 1170–1183, 1986.

Summary + Exercises

We started chapter 6 in the book of Wilkinson and Allen.

Exercises:

1. Write code using `MPI_sendrecv` in C or Python for a butterfly barrier. Show that your code works for $p = 8$.
2. Rewrite `prefix_sum.c` using `MPI_sendrecv`.
3. Consider the composite trapezoidal rule for the approximation of π, doubling the number of intervals in each step. Can you apply the prefix sum algorithm so that at the end, processor i holds the approximation for π with 2^i intervals?