The Golden Ratio*

CTTI

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1. In the rectangle below: $BD = ED$, $FE \parallel BD$ and $\frac{BD}{CD} = \frac{EC}{AC}$.

If $BD$ has length 1 and $CD$ has length $\phi$, compute $\phi$.

2. I used the letter $\phi$ to denote the ratio that holds when $\frac{a}{b-a} = \frac{b}{a}$ where $a$ and $b$ are the altitude and base of a rectangle. Do you know why? What are the advantages and disadvantages of this notation?

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3. Continue the diagram by choosing $G, H$ so that $CE = CG$ and $GH$ is parallel to $CD$.

So $\frac{BD}{CD} = \frac{EC}{AC} = \frac{AG}{AF}$.

(Verify this statement by calculating the lengths using what you know about $\phi$; it is easier if you use the equation $\phi^2 - \phi - 1 = 0$ instead of the actual value of $\phi$.)

4. What does the word commensurable mean? Can you think of synonym to explain for high school students. Is it part of the current geometry curriculum? It doesn’t appear in CCSSM; Should it?

5. Could it be the case that there are integers $m, n$ such that $\phi = \frac{m}{n}$. That is, is it possible that there are points $X$ on $BD$ and $X'$ on $CD$ such that $BX \cong CX'$ and $n$ copies of $BX$ make $BD$ and $m$ copies of $CX$ make up $CD$.

(Hint: Think of what this means for the successive rectangles, $ACBD, CEFA, GHFA$. More precisely, suppose $m_0, m_1, \ldots$ are the number of disjoint segments congruent to $BX$ in the long side of the $i$ rectangle. What can you say about the $m_i$? )

Note that the relation $\frac{m}{m-n} = \frac{1}{2}$ implies that in $CE$ is $m - n$ copies of $CX$ and $AC$ is $n$ copies of $BX$. That is at each stage the unit remains the same. So in fact the construction is guaranteeing that all sides of the successive rectangles are commensurable with the same unit.