Exploring Linear Equations

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Fisher Middle School
Algebra 1

Educational Transfer Plan – IISME Summer Fellowship 2005

Investigating Liner Equations Using Graphing Calculator

Overview:

Investigating Linear Equations unit consists of 6 graphing activities designed to help students explore and learn the concept of linearity using graphing calculator, an assessment rubric, and a self-evaluation form. These activities are not meant to substitute but supplement the chapter on graphing linear equations.

Time:

Each activity takes 1-2 class periods.

Requirements:

- Students should have the basic knowledge of pre-algebra math concepts, know about the four quadrants, ordered pairs, plotting points, and functions; know the meaning of slope and x- and y-intercepts; for activity #3, students should have mastered solving equations in two variables.
- Teachers should have the knowledge and skills of using the graphing calculators. Students learn best when teacher demonstrate how to use the graphing calculator.
- Teacher should prepare and give out handouts with instructions related to the specific calculator in use. For example, Texas Instruments, Casio or Sharp calculators are commonly used. In addition there is the Mackintosh graphing calculator whose program can be downloaded into the class computers. Below are two online references for TI-83 guides.
  - http://education.ti.com/us/product/tech/83p/guide/83pguideus.html
  - http://www.ncsu.edu/felder-public/kenny/papers/ti.html

Abstract:

The emphasis in this unit is on developing the slope-intercept form of the linear equation. In the first three activities, the student explores and learns about the slope and y-intercept of a line graph by graphing and seeing many graphs.
By activity 4 the student can write the equation of a given graph and check the result on the calculator. Activity 5 and 6 are real-world applications.
Activity 6 however applies the concept of linearity to scatter plot. students collect data of weight versus height of their family members. Make a scatter plot, and use the line of best fit to interpolate and extrapolate. The teacher guide the students in using the graphing calculator to check their work.

Objectives:

This unit is designed to help the tactile and visual learners in my inclusive 8th grade algebra class, learn and understand the concept of slope and linear equations, and how the concept of linearity is applied in real world situations.

Exploring Linear Equations

- Using the graphing calculator and interactive worksheet activities the student will load a function into the calculator and immediately see its graph.
- By graphing different lines, the student will see similarities and differences in the graphs that lead to specific conclusions based on what they see.
- By answering the questions on the worksheet, the student will compare, analyze, and make generalizations about the graph. .
- Providing these experiences, the student will begin to see how different coefficients (slope) affect the shape of the graph.
- By examining the graphs they see, students will identify the x- and y-intercepts of the graph.
- By looking at a line graph, the student will figure out the equation of the line in the slope-intercept form.
- Having students play with graphing calculators is fun. Students love fun. It makes learning a desirable rather than hateful experience.
- By using the graphing calculator students engage most of their senses in the learning process: touching with their fingers (kinesthetic), seeing (visual), thinking and reasoning (cognetive), and articulating their thoughts with their partners (auditory).
- Hence, by providing such experiences the teacher makes learning equitable to all students in the classroom.

Algebra 1 CA Standards:

- Standard 5.0 – Students solve multistep problems, including word problems, involving linear equations. (activities 3 and 4).
- Standard 6.0 – Graph a linear equation and compute the x- and y-intercept (activity 2).
- Standard 7.0 , 8.0 – Understand the slope for use in the point-slope formula and for understanding parallel lines and pependicular lines (activities 1, 2, 3).

National Board Standards:

- *NB Standard I - Commitment to All Students:*

"Accomplished mathematics teachers value and acknowledge the individuality and worth of each student; … they demonstrate these beliefs by providing all students equitable and complete access to mathematics".

- *NB Standard VI - Learning Environment*

“Accomplished mathematics teachers create stimulating, caring, and inclusive environments. They develop communities of involved learners in which students ….work independently and collaboratively, and value mathematics.”

- *NB Standard VIII - Technology and Instructional Resources:*

"Accomplished mathematics teachersare knowledgeable about and, where available, use current technologies and other resources to promote student learning in mathematics."

Relationship between ETP and my Fellowship:

- **Math in the work place at Intel**

- Working at Intel was a very rich and inspiring experience. I was a member of a team working on optimizing WIP (Work In Progress) delivery to stations based on process flow and geographical location of work stations in the Factory (Factory Layout). This kind of work involved a good knowledge of math in both analyzing statistical data and reading and intrerpreting graphs. So, I chose a topic for my ETP related to graphs – graphing linear functions, to teach students the concept of linearity between two variables; a topic that most students find difficult to understand in the middle school.

- **Technology in the work place at Intel**

Technology becomes an essential part of learning now-a-days. Hardwares, softwares, automated machines, all are incorporated to enhance learning, training, communication, research, development, production, and sales at Intel. When technology is used to aid instruction in the classroom it enhances the teaching proces and optimises learning because it will reach not only the auditory learner but also the visual and tactile learner. So, I chose to use the technology inbeded in the graphng calculator so students play with equations and see the result on their graphs.

- **Well structured programs for optimal yield at Intel**.

As a company, Intel is highly structured and this is reflected in its programs and activities. I chose well structured interactive activities for my ETP to help my students learn the concept of linear relationships step by step and understand the process and its reasoning before drawing conclusions.understanding the process will ensure good results.

- **On going research and development thriving for Excellence.**

Intel engineers and administrators continuosly monitor and assess work in progress to assure high quality production and optimal results. they invest in research as a means for development: a pathway to excellence.
So by continually monitoring and assessing students learning and achievement and working on areas that needs developing we can optimise the outcome of our teaching.

Materials/ Resources Needed:

- Graphing calculator
- Graphing calculator guide/ teacher instructions handout
- Student's Algebra Text Book: Prentice Hall, McDougal Littell, or other.
- Graph papers

Sources:

- Texas Instruments TI-83 plus Explorations (http://www.ti.com/).
- TI-83 plus Instructions booklet.
- Graphing Calculator Activities – Dale Seymour.
- Prentice Hall Algebra 1.
- Navigating through Algebra – NCTM.
- Math Forum - Ask Dr. Math.url
- http://www.teach-nology.com/web_tools/rubrics/
- http://rubistar.4teachers.org/index.php

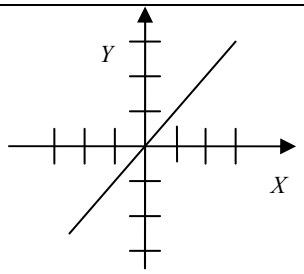
Name ----------

Activity 1 Date ---------- Per ----

Graphing Lines of the Form *y = mx*

Objective: In this lesson you will see how the value of m affects the graph of a straight line.

1. Use the graphing calculator to graph each function and complete the table. An example is done for you.

Function	Value of *m*	Sketch	*x*-intercept	*y*-intercept	Is the graph the same, steeper, or flatter than the graph of *y = x* (or *y = -x*)?
y = x	1		(0,0)	(0,0)	The same as *y = x*.
y = -x					
y = 2x					
y = -2x					
y = 0.5x					

Exploring Linear Equations

Function	Value of *m*	Sketch	*x*-intercept	*y*-intercept	Is the graph the same, steeper, or flatter than the graph of $y = x$ (or $y = -x$)?
$y = -8x$					
$y = -0.4x$					
$y = 0.05x$					

2. Use the results to answer the following items.

a. What point does every graph in exercise 1 have in common? _____

b. Does the value of *m* affect the location of the *x*- or *y*-intercept? _____

c. If the graph lies in quadrants I and III, then *m* is (positive, negative). Circle one answer

d. If the graph lies in quadrants II and IV, then *m* is (positive, negative). Circle one answer

e. If *m* is positive, predict what will happen to the graph as *m* gets larger. _____

_____

f. If *m* is negative, predict what will happen to the graph as |*m*| (absolute value of *m*) increases.

_____

g. Make a conjecture about the graph when $m = 0$.

_____

_____

Check your prediction by graphing the function on your calculator.

h. Summarize the role of *m* in the graph of $y = mx$. _____

_____

_____

_____

3. For each of the following, write an equation of a line that fits the characteristics. Verify your answers with the graphing calculator.

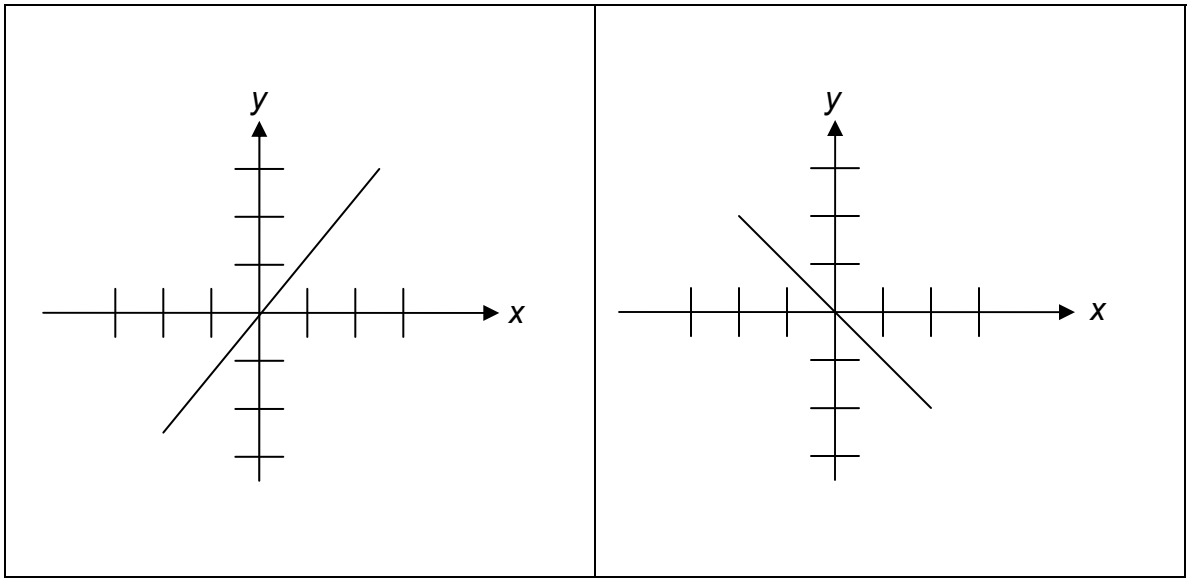
a) A straight line in quadrants I and III steeper than $y = 3x$.

Equation: _____

b) A straight line in quadrants II and IV flatter than $y = -x$

Equation; _____

c) A straight line with the graphs shown below.



Equation: _____ Equation: _____

d) A straight line that lies between the graphs of $y = 3.5x$ and $y = 1.5x$.

Equation: _____

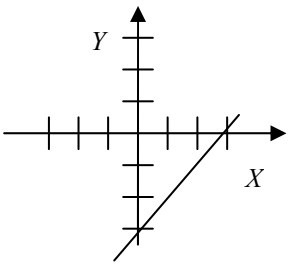
Name ----------
Date ---------- Per -------

Activity 2

Graphing Lines of the Form $y = mx + b$

Objective: In this lesson you will see how the constant b affects the line graph.

1. Use a graphing calculator to graph each equation and complete the following chart. An example is solved for you.

Equation	Value of *m*	Value of *b*	Sketch	*y*-intercept	*x*-intercept
$y = x - 3$	1	-3		(0,-3)	(3,0)
$y = x + 4$					
$y = x + 5.5$					
$y = 2x - 5$					
$y = 2x + 4.8$					

Exploring Linear Equations

Equation	*m*	*b*	sketch	*y*-intercept	*x*-intercept
$y = 3x - 2$					
$y = -3x + 7$					
$y = -3x$					

2. Use the results to answer the following questions.
   a. If *b* has a positive value, then the *y*-intercept is (above, below) the *x*-axis. Circle one answer.
   b. If *b* has a negative value, then the *y*-intercept is (above, below) the *x*-axis. Circle one answer.
   c. What is the *y*-intercept of the equation $y = 2x + 4$? _____
   d. What is the *y*-intercept of the equation $y = mx + b$? _____

3. Answer the following questions about the first three entries in Exercise 1.
   a. What is the same about all three graphs? _____
   b. These lines never intersect so we say they are _____
   c. What is the relationship between *b* and the *x*-intercept in these equations?

_____

   d. What are the *x*- and *y*-intercepts of $y = x - 5$? _____
   e. How does changing the value of *b* affect graphs of the form $y = x + b$? _____

_____

4. Describe and compare the graphs of $y = 3x - 1$ and $y = 3x + 2$. (Use a graphing calculator to help you see the graphs.) _____

_____

_____

5. Write an equation whose graph is a horizontal line. _____
6. Write an equation whose graph is a horizontal line through (0, 2.5). _____
7. Write an equation whose graph is a line parallel to and between the graphs of $y = 3x + 2$ and $y = 3x + 4.5$ _____
8. Write and equation whose graph is a line parallel to the graph of $y = -3x + 1$, but with *y*-intercept (0,-5). _____

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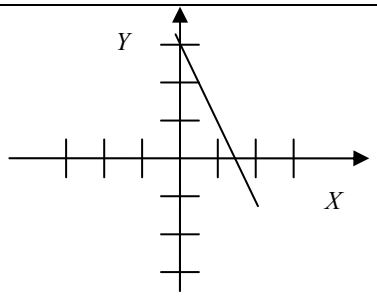
Activity 3

Graphing Lines

Objective: *In this lesson you will see how to graph equations that are not in the slope-intercept form y = mx + b.*

Solve each equation for y then write the equation in slope-intercept form. Find the slope, and x- and y-intercepts, and graph the line. An example is solved.

Equation	Equation in slope-intercept form	sketch
2x + y – 3 = 0 Solve for y: 2x + y – 3 = 0 -2x = -2x Y – 3 = -2x + 3 = + 3 Y = -2x + 3	Y = -2x + 3 Slope = -2 y-intercept =(0,3) x-intercept = (1.5,0)	
Y + 3x = 4		
Y – 3.5 = 2x		
5x – y = 15		

Exploring Linear Equations

Equation	Equation in slope-intercept form	sketch
-1x = 4 + y		
2y + 5x – 7 = 0		
***Challenge*:** Write your own problem below. Follow the directions above to solve and graph the line.		

Name ----------

Activity 4 Date ---------- Per -------

Find That Equation

Objective: *In this lesson you will see how to find the equation of a line by looking at its graph.*

Examine each graph below and predict its equation. Then use the graphing calculator to test your prediction. The first problem is solved for you.

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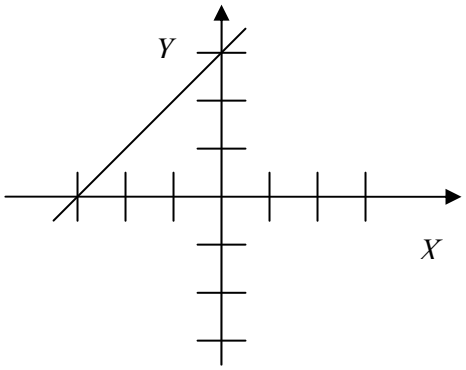
<td> 
</td>
<td>Reasoning:
The y-intercept is (0, 3). -----→ b = 3
The slope is positive and equals 1. ----→ m = 1
The slope-intercept form -----→ y = mx + b
Substitute the values of m and b.
y = 1x +3

Equation: y = x + 3</td>
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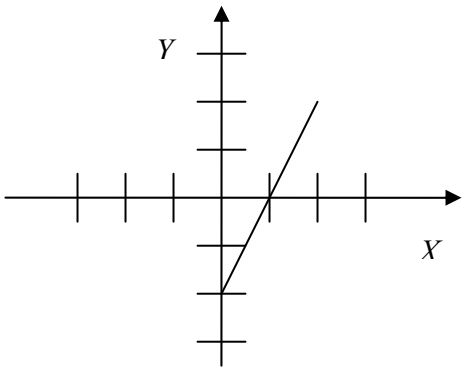
<td> 
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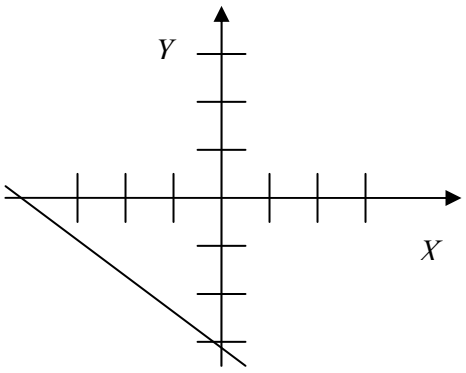
<td> 
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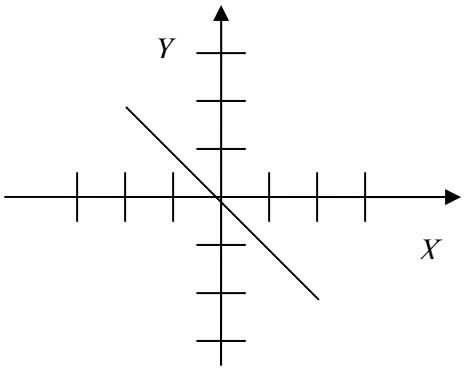
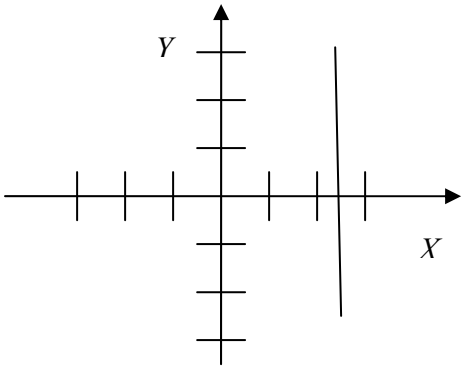
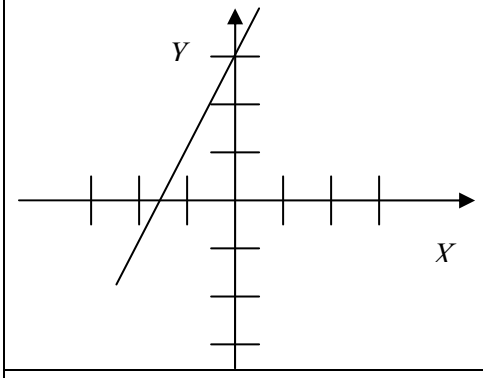
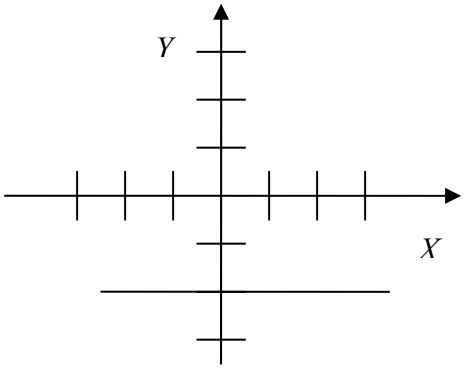
Exploring Linear Equations

Name ----------
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Activity 5
Real-World Application
Pledge Plans

Objectives:

- *Apply the concept of linear equations solve real-world problem*
- *Use a table to organize information*
- *Make a graph to display data using heading, labels, and scales*
- *Use a graphing calculator to see if the graph is correct*
- *Recognize the relationship among the tables, the graph, the equation, and the slope of the line*
- *Identify the y-intercept from a graph or a table*

Read the problem carefully and answer all the questions.

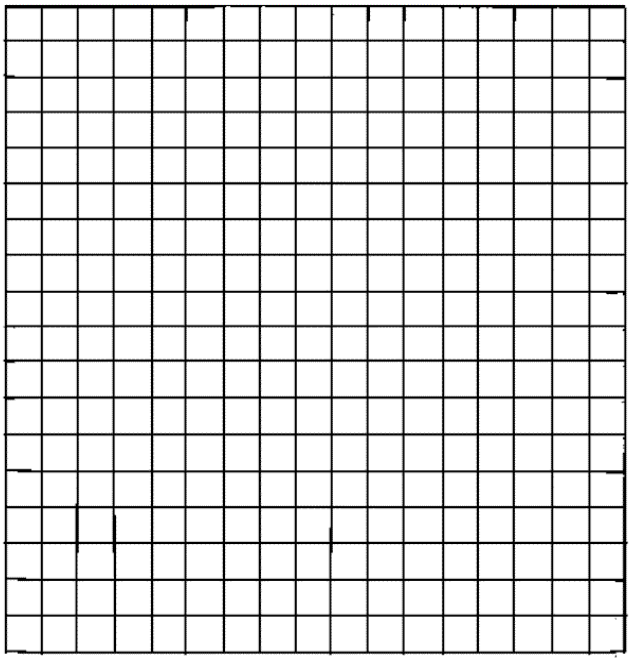
Several students from "Help Your School Club" want to raise money for your school. They are participating in a 10-Kilometer walk-a-thon and need to decide on a plan for sponsors to pledge money for the walk-a-thon. Chris thinks that $1.50 per kilometer would be an appropriate pledge. Natalie suggests $2.50 per kilometer because it would bring in more money. Grace says that if they ask for too much money, people won't agree to be sponsors; she suggests that they ask for a donation of $4.00 and then $0.75 per kilometer.

1. Fill in the table below showing the amount of money a sponsor would owe under each of the pledge plans. The dollar amounts for 0 Km and 1 Km are filled in as examples.

Distance	**Money Owed (Dollars)**		
1-10 Km	**Chris**	**Natalie**	**Grace**
0	0.00	0.00	4.00
1	1.50	2.50	4.75
2			
3			
4			
5			
6			
7			
8			
9			
10			

Use the grid below to answer question #2.

2. On the same coordinate grid, make a graph for each of the three pledge plans. Use a line to connect the points. Remember to give your graph a title, label your axes, and show the scale on each axis.



3. Use graphing calculator to check your graph.

Suggested Window
Xmin = 0
Xmax = 10
Xscl = 1
Ymin = 0
Ymax = 25

4. For each of the three plans, describe in your own words, the relationship between the money earned and the distance walked.

5. Write an equation that can be used to compute the money owed under each pledge plan. Use *M* to represent the money owed and *d* to represent the distance the student walks.

6. State the slope and the y-intercept for each of the above equations.

7. Describe how increasing the amount of the pledge per kilometer affects the table, the graph, and the equations.

8. Does the amount of the pledge per kilometer in question #7 relate to the slope of the line? How does it relate? Explain.

9. Describe what is different about Grace's plan. What happens in the table, the graph, and the equation when Grace's plan is introduced?

Challenge:

10. Write your own pledge plan and sketch its graph. Make your own axes and show the labels and scale. Use the graphing calculator to check your graph.

Activity 6
Scatter Plot and Line of best Fit

Name: ----------
Date: ---------- Per. ------

Objectives:

- *Using the graphing calculator, graph a scatter plot of two related variables.*
- *Graph the" line of best fit".*
- *Interpolate and extrapolate points in your data.*

Collect data of the height (in inches) and the weight (in pounds) of each member in your family. Make a table of your data and bring it to math class (At least 8 entries).

1. Make a scatter plot of your data on a grid paper. Remember to show a title, label and scale on each axis. Draw the line of best fit.

2. Use the graphing calculator and graph the scatter plot for your data.

3. Follow the graphing calculator manual or your teacher's instruction to show the line of best fit on your calculator's screen.

4. Compare the graph of the calculator with your graph on the grid paper. Are they the same? _____. Are they different? Why? _____

5. Use the "trace" feature and the line of best fit to interpolate and extrapolate the weight and height of an absent (or a virtual) family member. Examples of what you need to find are explained below.

Interpolate:

What would be the weight of a family member whose height is in between the shortest and the tallest of your family? (The point is somewhere inside your data)

Extrapolate*:*

What would be the weight of a family member whose height is shorter than the shortest, or taller than the tallest family member? (The point is somewhere outside your data).

6. Use the "linear regression" feature of the graphing calculator to find the slope and y-intercept of the line of best fit.

7. Write the equation of the line of best fit. _____

Challenge:

Show on a piece of paper, how you would use the slope and y-intercept of the line of best fit to interpolate and extrapolate the height and weight of the two virtual family members in question #4. Did you get the same result?

Exploring Linear Equations

Mrs. Alsumidaie

Name ----------

Title of Work ---------- Date submitted ---------- Per –

Criteria					Points
	4	**3**	**2**	**1**	
Assignment Completeness	All items attempted.	90% of items attempted.	At least 50% of all items attempted.	Less than 50% of all items attempted.	____
Accuracy	All items are correct.	90% of items are correct.	Between 50% and 90% of items are correct.	Less than 50% of all items are correct.	____
Graph Neatness	All sketches / graphs are neatly drawn. All lines are straight, and all axes are labeled with correct scale showing.	90% of graphs are neatly drawn, with straight lines, labeled axes and correct scale showing.	At least 50% of graphs are neatly drawn, with straight lines, labeled axes and correct scale showing.	Less than 50% of graphs are neatly drawn, with straight lines, labeled axes and correct scale showing.	____
Demonstrated Knowledge	Shows complete understanding of the questions, mathematical ideas, and processes.	Shows substantial understanding of the problem, ideas, and processes.	Response shows some understanding of the problem.	Response shows a complete lack of understanding for the problem.	____
Requirements	Goes beyond the requirements of the problem.	Meets the requirements of the problem.	Does not meet the requirements of the problem.		____
Legibility	Legible handwriting, typing or printing.	Marginally legible handwriting, typing, or printing.	Writing is not legible in places.	Writing is not legible.	____
				Total --→	

Teacher's comments: _____

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1. What did you learn from these activities?

2. What did you like best about these activities?

3. What was most difficult about these activities?

4. What would you do differently next time?

5. What do you think your teacher should do differently next time?