Jan 12. Systems of equations and graphing with calculator

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name cards
The 4-fold way

1. verbal
2. symbolic
3. graphic
4. tabular

How are these related? Do you know how to use each approach?
Some Answers

From a verbal description of a situation we choose a symbolic representation of the problem.

A symbolic representation determines the table and the graph.

A finite table or graph *suggests* a function.
Graphing Calculators

How much experience do you have with graphing calculators? Graph on the calculator.

\[ y = 7x - 9 \]

\[ 2x - 3y = 4 \]
Solutions of systems of linear equations

Review: What is a solution of

\[ ax + by = e \]  \hspace{1cm} (1)
\[ cx + dy = f \]  \hspace{1cm} (2)
Some possible answers

1. The collection of pairs \((u, v)\) such that:

\[
a u + b v = e \tag{3}
\]
\[
c u + d v = f \tag{4}
\]

2. The points where the lines

\[
a x + b y = e \tag{5}
\]
\[
c x + d y = f \tag{6}
\]

intersect.
Exact Solutions

Find the exact solution of

\[ y = -x + \frac{2}{17} \]  
\[ y = x \]
Exact ???

What do you know about rational, irrational and transcendental numbers?
What do you know about rational, irrational and transcendental numbers?

Exact solutions. answer must be
1/11 not .09999999999 or
π not 3.14159...
What’s so great about linear equations?

Why do we emphasize linear equations?
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Extrapolate
Interpolate
What’s so great about linear equations?

Why do we emphasize linear equations?

Extrapolate
Interpolate

wikipedia says:
In mathematics, extrapolation is the process of constructing new data points outside a discrete set of known data points. It is similar to the process of interpolation, which constructs new points between known points, but the results of extrapolations are often less meaningful, and are subject to greater uncertainty.
Linearity vrs Proportionality

What is the difference?
Linearity vrs Proportionality

What is the difference?

Linear: \( y = ax + b \)
proportional: \( y = ax \)
Data Analysis

Given some collection of data, we would like to find an explanation for the relations among the data. Ideally, we
WRITE AN EQUATION THAT ‘AGREES’ WITH THE GIVEN DATA.
1. There should be a theory, an explanation, of why this kind of function is appropriate for this kind of data. In particular, we must be able to describe in words the meaning of the coefficients and specify their units.

2. We may try to ‘curve-fit’ as a means of conjecturing the correct explanation. But it is essential to find an explanation for why the equation has the conjectured form.
Three Strategies

We will decide on a model and try to find the coefficients for our data.

1. spaghetti
2. linear regression: $ax + b$
3. proportional regression: $ax$
Some examples

1. view tube
2. Water Drops
3. Earth gravity
4. Mars gravity
5. Hubble’s constant
Each group has a data set in which the relationship is known to be of the form $y = mx$.

What does $m$ mean in your problem?

Estimate the value of $m$ using

1. spaghetti
2. linear regression: $ax + b$
3. proportional regression: $ax$
Complexities

Anscombe’s data sets:
anscombe.doc
http://exploringdata.cqu.edu.au/anscomb2.htm
What’s linear about this?

badmath.doc
Vocabulary

Exact Solutions
linear vrs proportional
extrapolate vrs interpolate
curve fitting – regression