March 4: Graphing Quadratic polynomials

John T. Baldwin

March 5, 2009

Next Quarter

The class next quarter will be on 10 Monday nights same time (5:00 PM -8:15 PM) same place (Munroe School)
starting Monday, March 30 and ending Monday, June 8. Class will not be held on Memorial Day, Monday, May 25 or Monday April 6.

Some transformations

Work on and discuss the IMP handout
The ups and downs of quadratics

What do transformations do?

What did we have to do to

1. move graph up?
2. move graph down?
3. move graph to left?
4. move graph to right?
5. widen or narrow with same vertex?

Standard form

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$$\begin{array}{rc} ax^2 + bx + c & \text{polynomial normal form} \\ a(x-h)^2 + k & \text{vertex normal form} \end{array}$$

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$ax^2 + bx + c$	polynomial normal form
$a(x - h)^2 + k$	vertex normal form
$a(x - r_1)(x - r_2)$	root normal form

Geometric properties of parabolas

What are

1. axis of symmetry?
2. roots?
3. vertex?
4. minimum/maximum

The standard quadratic

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$$y = x^2 + 2x + 1$$
$$y = x^2 - 3x + 2$$

Roots and Axis of Symmetry

Look at CME: 713; 720-723

Determining Equations

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No! This is why root normal form, $a(x - r_1)(x - r_2)$ has an a in it.

Homework analysis

1. Writing; when does the and appear? What is the logic of the solution? Any x satisfying the inequality is blah3 and blah4 or blah1 or blah2.
2. How many terms in the product of two trinomials?
3. $d = \frac{at^2}{2}$
4. CME 641 2a, 2b. What is the difference?

What is a written solution of an equation/inequality

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It is a series of deductions about any number(s) that might satisfy the

1. equation
2. inequality
3. system of equations
4. system of inequalities

Writing inequalities

The next three slides represent three ways to write down a solution to some inequalities involving absolute value. Only the last gives a clear indication of the logical flow of the solution. The others are procedures.

Writing inequalities

$$|3x - 4| < 9$$

$$\begin{aligned} 3x - 4 &< 9 \\ 3x &< 13 \\ x &< \frac{13}{3} \end{aligned} \qquad \begin{aligned} 3x - 4 &> -9 \\ 3x &> -5 \\ x &> \frac{-5}{3} \end{aligned}$$

$$|4 - 7x| \geq 16$$

$$\begin{aligned} 4 - 7x &\geq 16 \\ -7x &\geq 12 \\ x &\leq \frac{-12}{7} \end{aligned} \qquad \begin{aligned} 4 - 7x &\leq -16 \\ -7x &\leq -20 \\ x &\geq \frac{20}{7} \end{aligned}$$

Writing inequalities

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$$\begin{array}{rcl} 3x - 4 & < & 9 \\ 3x & < & 13 \\ x & < & \frac{13}{3} \end{array} \quad \text{and} \quad \begin{array}{rcl} 3x - 4 & > & -9 \\ 3x & > & -5 \\ x & > & \frac{-5}{3} \end{array}$$

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Writing inequalities

For any real number x, each sentence implies the next.

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Mini-max problems

Turn to CME page 703.

Problem 3 page 703

Let h be the height and w the width. Then $w = 100 - 2h$. We want to minimize $A = h(100 - 2h)$.
So $A = 100h - 2h^2$; we rewrite it as

$$A = -2(h^2 - 50h).$$

Then we complete the square to $-2(h^2 - 50h + 625) + 1250$.
So in vertex normal form:

$$A = -2(h - 25)^2 + 1250.$$

So the maximum is attained when $h = 25$, $w = 50$ and the area is 1250 square feet.

Antonia's observation

Instead of completing the square, in the last problem we could consider the function:
$A(h) = 100h - 2h^2$.
The maximum of that function will be attained when h is on the axis of symmetry. And we noted in our earlier discussion (CME page 721) that for any quadratic equation $ax^2 + bx + c$, the axis of symmetry is $x = \frac{-b}{2a}$.