Comments and Solutions to HW 4

1. Given the line L defined by the equation

$$2x + 3y = 7.$$

Put the equation in slope-intercept form: $y = -1.5x + 3.5$.

a) Then a line parallel to L is any line with same slope and different y-intercept, e.g. $y = -1.5x + .5$.

b) A line that intersects L at $(1, 2)$ is any line through that point. So I take random coefficients say 7 and 5. Then $7 \cdot 7 + 5 \cdot 2 = 59$ so $7x + 5y = 59$ is a correct answer. (So is $2x + 3y = 7$; it happens to intersect in more than one point since it is the same line.)

c) The only straight line that has more than one intersection with L is L. So any equation for L, e.g. $2x + 3y = 7$ or $4x + 6y = 14$ is correct.

Comment: The word 'other' in the problem statement is misleading. Taken seriously, it makes c) impossible since there is no 'other' line that meets L in at least two points.

2. This was routine. The only mistakes were silly arithmetic. For the record:

a) $x > -2.5$

b) $x \geq 0$

c) The problem is to solve the inequality

$$5x - 3 < 6x + 1/2.$$

A number of you did this correctly but followed a procedure that made the work harder and more mistake prone. (Overgeneralized rule: get x on the left. So $6x$ is subtracted from both sides giving a $-x$ and then one is led to divide by a negative number.)

The easiest way to solve this problem is to think entirely additively.

Subtract 5x from both sides to get

$$-3 < x + 1/2.$$

Subtract 1/2 from both sides to get

$$-3.5 < x$$

(which can be written $x > -3.5$).

This is an example of 'thoughtful manipulation'. Before diving in, I look at the coefficients and ask 'How can I avoid negatives?'. (In another context, I might ask, 'How can I minimize computations with fractions?')

This problem suggests another justification of 'dividing by a negative reverses the signs'.

Suppose I have $-A < B$. Add to both sides and subtract B from both sides. That gives, $-B < A$ (i.e. just reading the inequality in the other direction: $A > -B$), which is the way we think of the division rule.

3. Every paper I have graded so far has graphed the lines correctly. But several have identified the wrong region as the solution of the system of inequalities. This is easy to do and there is a simple technique to avoid that mistake. Choose a

point in the region you think is correct. Check by substitution that it satisfies each inequality. If it does, this is the correct region. If not, try a different region. The key concept here is that you can test any point in a particular region; if it satisfies all the inequalities then any point in the same region will satisfy all the inequalities. Choose one that makes the computation easy.

I have placed in a separate file (Assignment 4 graphs)on the webpage solutions to the linear inequalities in problems 3 and 4. The lines aren't labeled but they should be on your papers. (I don't have time to figure out the technology and this did not seem to be a problem for the class.)

4. Bob builds tool sheds. He uses 10 sheets of dry wall and 15 studs for a small shed and 15 sheets of dry wall and 45 studs for a large shed. He has available 60 sheets of dry wall and 135 studs. If Bob makes \$390 profit on a small shed and \$520 on a large shed, how many of each type of building should Bob build to maximize his profit?

Let x be the number of small sheds made and y the number of large sheds made. Then the problem imposes the following constraints:

$$\begin{cases} 10x + 15y & \leq 60 \\ 15x + 45y & \leq 135 \\ 0 & \leq x \\ 0 & \leq y \end{cases}$$

As in problem 3), solve this system of linear inequalites. The solution (feasible region) as pictured in the other file is a trapezoid with vertices: $(0,0),(0,3),(6,0),(3,2)$. Evaluate the profit function: $P(x,y) = 390x + 520y$ at each these four points. $P(0,0) = 0$, $P(0,3) = 1560$, $P(6,0) = 2340$, $P(3,2) = 2210$, so the maximum profit is obtained by building all small sheds.

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