Inequalities and quadratics

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Class outline

Exam

A: > 90
B: $80 < x \leq 90$
C: $70 < x \leq 80$
problem 6:
Linear regression on dog walk ?????
What is a piece of a function?

And versus Or

Suppose A and B are two statements.
When is A and B true?
When is A or B true?

And versus Or

Suppose A and B are two statements.
When is A and B true?
When is A or B true?

A and B is true exactly when both are true.
A or B is true exactly when at least one of them is true.
inclusive or.

Solutions of Absolute Value Inequalities

$|x| < a$ means
$x < a$ AND $x > -a$.

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$|x| < a$ means
$x < a$ AND $x > -a$.

$|x| > a$ means
$x > a$ OR $x < -a$.

Examples

$$|3x - 2| < 7$$

$$|4 - 5x| \geq 15$$

$$|-3x| = -1$$

Graphical Methods: Example

Example of the split point method: To solve $|3x - 2| < 7$ by the split point method, graph the two functions $y = |3x - 2|$ and $y = 7$. The solution is the set of numbers on the real line such the value of the first function is less than the value of the second.

The real axis is divided into a finite number of intervals by those x where the lines cross. These are called split points.

(In this example the intervals are $(\infty, -\frac{5}{3})$, $(-\frac{5}{3}, 3)$, and $(3, \infty)$. The middle one is where the inequality holds.)

Graphical Methods: General case

Let f and g be polynomials.
To solve: $f(x) < g(x)$.
Graph the two functions. They will cross at finitely many points a_i where $f(a_i) = g(a_i)$.
These are the split points. The solutions are the intervals determined by these split points where the inequality holds.

Two kinds of problems

1. inequalities in one variable. The solution is a union of intervals in the real line – a set of numbers.
2. inequalities in two variable. The solution is a set of points in the plane. The solution will be a shaded set of point in the plane.

Find the form $|x - a| < b$

Suppose we are looking at $\{x : -2 < x < 10\}$.

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$$|x - 4| < 6$$

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$$|x - 4| < 6$$

$|x - a| < b$
a midpoint of the interval; b is $1/2$ of the length.

Graphs I

Discuss problems 3, 4, 5 from the interpreting graphs homework.
Velocity versus speed.

Why study quadratics

1. Equations:
   A rectangle is two feet longer than it is wide. If the area of the rectangle is one square foot, what are the dimensions of the rectangle?

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1. Equations:
   A rectangle is two feet longer than it is wide. If the area of the rectangle is one square foot, what are the dimensions of the rectangle?
   Functions: A ten pound weight is dropped from the leaning tower of Pisa. After 9.3 seconds it hits the ground. How fast is it going when it hits? How high is the Leaning tower of Pisa?

Distance $=$ rate times time

What does this mean?

Distance = rate times time

What does this mean?

constant rate

Acceleration

A ball rolls down a ramp with a constant acceleration of 4 ft. per second squared.
What is its velocity after 1 second, 2 seconds etc.
How far has it traveled after 1 second, 2 seconds etc. ?

velocity = acceleration times time

consider examples.

$$v = at$$

velocity = acceleration times time

consider examples.

$$v = at$$

What was the average velocity?

velocity = acceleration times time

consider examples.

$$v = at$$

What was the average velocity?

$$v = \frac{at}{2}$$

Why? Velocity at beginning is 0. Velocity at end is at.

So the average is $\frac{0+at}{2} = \frac{at}{2}$.

velocity = acceleration times time

consider examples.

$$v = at$$

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What is the area under the line?

distance

Constant acceleration; initial velocity 0:

$$d = rt$$

So we use the average velocity we computed on the previous slide as if it were a constant velocity for the problem.

$$\begin{aligned} d &= \frac{at}{2}t \\ &= \frac{at^2}{2} \end{aligned}$$

Example

A ball rolls down a ramp with a constant acceleration of 6 ft. per second squared.
a) What is its velocity after 5 seconds?
b) How far does it travel in 5 seconds?

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A ball rolls down a ramp with a constant acceleration of 6 ft. per second squared.
a) What is its velocity after 5 seconds?
b) How far does it travel in 5 seconds?

c) If the ramp is 10 feet long, when does the ball reach the bottom of the ramp?

Solution

a) $v = at$, so $v = 6 \times 5$; 30 feet per second.
b) The average velocity is the final velocity over two: 15 feet per second. So the distance traveled is 15×5; 75 feet.
c) $d = \frac{at^2}{2}$ and $a = 6$ while $d = 10$. So $\frac{6t^2}{2} = 10$ and $t = \sqrt{(}\frac{10}{3}$.

Some terminology

A monomial is a product of numbers and variables. e.g.

$$x,$$

$$3x^2,$$

$$2xy^2$$

$$\frac{3}{4}s^2x^3z.$$

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A polynomial is a sum of monomials.
binomial, trinomial

multiplying polynomials

What is

$$(a+b)(c+d)?$$

multiplying polynomials

What is

$$(a + b)(c + d)?$$

What is

$$(a + b)(c + d + e)?$$

Multiplying binomials

$$(x+y)(x-y) = x^2 - y^2$$

$$(x+y)(x+y) = x^2 + 2xy + y^2$$

$$(xy)^2 = x^2y^2$$

Algebra and Arithmetic

Multiply in your head. $18 * 22$, $99 * 101$, 40^2, $41 * 39$.