A Drop of Water A Lesson for Sixth, Seventh, and Eighth Graders<br>By Jennifer M. Bay-Williams and Sherri L. Martinie<br>From Online Newsletter Issue Number 16, Winter 2004-2005

In Walter Wick's picture book A Drop of Water: A Book of Science and Wonder (New York: Scholastic, 1997), beautiful photographs show what can happen with soap bubbles when different objects are dipped in soapy water. Here, students blow bubbles onto their desks. When the bubbles pop, they leave residue in the shape of a circle, and students use the circles to investigate relationships among the radius, diameter, and circumference of circles. This lesson appears in the new book Math and Literature, Grades 6-8, by Jennifer M. Bay-Williams and Sherri L. Martinie (Sausalito, CA: Math Solutions Publications, 2004).

After reading A Drop of Water aloud, I drew on the board a large blank four-
 column table for recording data. I labeled the first column Circle Number and then asked what a diameter of a circle was.

Ariella said, "It goes all the way across the circle."
Madison added, "It has to go through the center."
I labeled the second column Diameter and then asked if they knew what a radius was. Lucas drew a circle on the board, and then drew a radius on the circle. I labeled the third column Radius.

Finally, I asked for someone to tell me what a circumference was. I labeled the fourth column Circumference.

| Circle <br> Number | Diameter | Radius | Circumference |
| :--- | :--- | :--- | :--- |
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|  |  |  |  |

I explained that today we were going to look at circles made by bubbles. We were going to look at many different-sized circles and measure the diameter, radius, and circumference of each one. Our goal was to find relationships among these measurements. I explained that they were going to use bubble mix to create bubbles on their desks.

I organized students into groups of four. Each student would have materials to blow his or her own bubbles, but they were to collect the data as a group. I wrote the directions for blowing bubbles on chart paper.

1. Place a small puddle of bubble mix on your desk.
2. Place a straw into the puddle.
3. Blow slowly into the straw.
4. Stop when the bubble is the size you want and pop it, or stop when the bubble pops by itself.
5. Measure across the circle to get the diameter and the radius and record on your group's table. The diameter will be where the line across the circle is the widest.
6. Use string to wrap around the circle and find the circumference and record on your group's table.
7. Wipe the desk off and go back to Step 1, trying to create a different-sized bubble.

I called on Andre to demonstrate. He poured about a teaspoon of bubble mix onto his desk, poked a straw into the puddle, and blew. A bubble emerged and then popped. The circle left by the bubble turned out to be 5 inches across, so I wrote 5 inches in the chart on the board under Diameter. Students knew that the radius would be half that and said the radius would be 2.5 inches. "What about the circumference?" I asked. Markita suggested they wrap a piece of string around the circle and then measure the string's length.

I explained that they would need to measure quickly, because the bubble outlines wouldn't last long. I encouraged them to start small, blowing circles that were around 2 to 4 inches in diameter. I gave each group a tray that had a plastic cup half full of water, a bubble jar, four straws, several paper towels, and four pieces of string. I allowed five minutes for them to practice blowing bubbles on their desks and then asked them to stop blowing bubbles and to wipe off their desks. I explained that each group needed to develop a table like the one on the board and record the data from each person in the group. "To start, please make smaller circles. Record your information and look for patterns," I explained.

As I circulated, I noticed students were intrigued by what they were finding.
"Look! The diameter is one-half inch, the radius is one-fourth inch, and the circumference is three inches," Marcus said about his first circle.

Lydia stretched out her string after she had found the circumference and announced, "Look at my circumference, it's eight inches!"

Students also discussed how to measure accurately. "How are you measuring circumference?" Rebecca asked Lindsey.

Lindsey explained, "Start at the end of the string and go around and mark the place where it ends."

After about ten minutes, I said, "Now you can try to make large circles." Students were excited to create the biggest bubble, and they carefully measured and recorded each circle they made.

After another ten minutes, I asked students to put their supplies away and clean off their desks. I explained that each group now needed to study its data. "I want you to analyze it as
mathematicians. Are there any patterns? Does any of your data look like it doesn't belong? What could have happened in your measuring that might have affected accuracy?"
"It's hard to see exactly where your finger was on the string," Rebecca explained.
"When you're measuring, it's hard to see where the exact center is," Lucas added.
"Keep that in mind as you look at your data. You might see small errors, which is OK, but keep your eyes open for data that doesn't fit and consider why you think that happened. I'm going to give you five minutes to look at your data. Then be ready to share any patterns you notice, errors you found, and what you think caused those errors," I directed.

As students worked, I roamed around and observed the discussions. One group discussed the diameter-to-circumference relationship. Andre had already written $\times 2$ or $\times 4$ or $\times 3$ next to each number in the Circumference column, estimating about how many times bigger than the diameter each one was.
"It looks like it's about four," Andre hypothesized, as he pointed at a few rows where he had recorded $\times 4$.

Alyssa said, "Yeah, but others are times three or times two."
"I think it might be pi," Jason suggested.
Andre responded, "If you look at all of these, it does look like three is about average." At that, they began using their calculators to find more exact ratios between diameter and circumference.

Another group wasn't familiar with pi but had more accurate measures than the previous group. They figured out that the circumference nearly always turned out to be a little more than three times the diameter. I asked the group if they thought their rule would work even for large circles. "Think of a hula hoop," I said. They weren't sure. We had a hula hoop in class and measured a longer string that went across the diameter three times. Together, we held the string around the hula hoop and found that it didn't quite make it all the way around.

Ariella said, "That is what happened in our table-three diameters isn't quite enough to equal the circumference."

Another group discussed the patterns:
"Circumference is diameter times two," Randy noted.
"Where did the two come from?" Micaylah asked.
Randy responded, "I tested it a couple of times and it works."
Micaylah still wasn't convinced and said, "Two doesn't seem right." A teammate pointed at two specific rows in the table where the circumference was approximately twice the diameter. Micaylah responded by pointing at other columns that were "times three" and "times four." The group continued to search for the most accurate multiple.

Alyssa added two more columns to her table. She explained, "I added a column titled 'C divided by $\mathrm{D}^{\prime}$ because I wanted to find the relationship between these so I wouldn't always
have to measure both of these." Her other column was titled $C \div R$. Again, she reasoned that this would help her find a pattern. All the other groups had settled on "about three times" as the relationship from diameter to circumference. Wanting them to be more specific, I asked Alyssa to share her strategy with the class. She did, and other groups began using division as a strategy to find a more exact relationship between diameter and circumference and radius and circumference.

After another five minutes, I added the new columns to the class table and asked each group to give data for one of its circles. This is the data we collected:

| Circle <br> Number | Diameter | Radius | Circumference | $C \div D$ | $C \div R$ |
| :---: | :---: | :---: | :---: | :---: | :--- |
| 1 | 3.5 | 1.5 | 10 | 2.9 | 6.7 |
| 2 | 5 | 2.5 | 15.7 | 3.14 | 6.28 |
| 3 | 7.5 | 3.75 | 19 | 2.53 | 5.07 |
| 4 | 3 | 1.5 | 9 | 3 | 6 |
| 5 | 6 | 3 | 20.3 | 3.4 | 6.8 |
| 6 | 17 | 8.5 | 50 | 2.94 | 5.88 |

After recording this data, I asked the students what patterns they noticed. Randy stated that the radius is always half the diameter.
"The first circle in the table doesn't fit the rule," Rebecca pointed out.
Andre added, "Yeah, well you can't be exact with the string and ruler, and that's close."
I asked, "How would I figure out the radius if I knew the diameter? How would I find the diameter if I knew the radius?" Students explained that to get the diameter you double the radius, and to get the radius, you take half of the diameter.
"What about circumference?" I asked the class.
Madison said, "Multiply diameter by pi."
Only a few students in the class knew what pi was, so I responded, "Assume I don't know what pi is-can you tell me the relationship you found in your data?"

Madison explained, "Well it's about three times the diameter." I asked her where she was getting that information. She said she was looking at the $\mathrm{C} \div \mathrm{D}$ column. Then she added, pointing at the class chart, that some results were more than 3 and some were less than 3 , but they should all be 3.14.

Since a student had mentioned pi, and had said it was 3.14, I decided to discuss this approximation of pi. I asked, "What does the 'point fourteen' in pi mean? Is it a lot?" Ariella said that it was like 14 out of 100 , which isn't much, so it fit with being slightly more than 3 .

I then had students choose from a collection of circular objects. I directed them to measure the diameter and multiply it by 3.14 to find the circumference. After multiplying, they used a new piece of string to measure the circumference and see how the two measurements compared.

I then asked how to use one of a circle's measurements to find the other two. I asked, "If I only have enough string to find the radius, how can I determine the diameter? The circumference?" For their written assignment, I asked them to describe all the relationships they had figured out, for example:

- circumference to diameter
- diameter to circumference
- radius to diameter
- diameter to radius
- pi to diameter

