

Geometry and
Proof

John T.
Baldwin

Background

Hilbert's
Critique

Three
Frameworks

High School
Curriculum

Geometry and Proof

John T. Baldwin

May 6, 2007

My background

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- 1 Model theory research (35 years)
- 2 working with teachers and future teachers (20 years)

Origin of This Talk

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- 1 six sessions with high school teachers on 'how to teach geometry'
- 2 One session of History of Mathematics on 'the superposition principle'

Euclid, Hilbert (google)

Hartshorne, Weinzweig

Solomonovich: review of modern books and introducing his

<http://www.solomonovich.com/geometry/textbook.html>

Raimi: Why the 'New Math' brought algebra into geometry

<http://www.math.rochester.edu/people/faculty/rarm/igno.html>

PROOF?

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[http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-829637-](http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-829637-4&chapter=2&lesson=7&quizType=1&headerFile=6&state=il)

[4&chapter=2&lesson=7&quizType=1&headerFile=6&state=il](http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-829637-4&chapter=2&lesson=7&quizType=1&headerFile=6&state=il)
(You have to change slash & back to just ampersand to get the site.) or just google glencoe. Why does it take six steps to show:

If two line segments have the same length and equal line segments are taken away from each, the resulting segments have the same length.

The remainder of the talk is a discussion of why geometry texts in the U.S. came to be that way.

State Goals

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Go to Goal 9 of Illinois State Standards.

See geometry standards at

<http://www.isbe.state.il.us/ils/math/standards.htm>

CONTEXT

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How does the axiomatization of geometry affect the teaching of high school geometry?

Logical Argument vrs 'Argument'

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Logic analyzes the 'soundness' of an argument.
Do true premises lead to true conclusions?

Logical Argument vrs 'Argument'

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Logic analyzes the 'soundness' of an argument.
Do true premises lead to true conclusions?

Checking the truth of the premises is

- 1 Mathematics if the premises are mathematical.

Logical Argument vrs 'Argument'

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Logic analyzes the 'soundness' of an argument.
Do true premises lead to true conclusions?

Checking the truth of the premises is

- 1 Mathematics if the premises are mathematical.
- 2 Politics if the premises are political.

Hilbert's Critique

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- 1 Undefined Terms
- 2 Continuity Axioms
- 3 The Mobility Postulate

CONTEXT

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Look at Euclid's definitions.

Can you distinguish two different types of definitions in this list?

Undefined terms

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Two kinds of definitions:

- 1 The 'system' of basic notions, not the individual notions, (points, lines, etc) is defined.
- 2 But auxiliary notions are introduced as abbreviations.

Continuity Axioms

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The continuity axioms leads to 'geometry over the reals'.
'Coordinatizing Ring' is a foreign notion to the Greeks.

Continuity Axioms

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The continuity axioms leads to 'geometry over the reals'.
'Coordinatizing Ring' is a foreign notion to the Greeks.

How do you explain similarity of figures whose side lengths are
incommensurable?

Superposition Intuition

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Common notion 4

Things which coincide with one another equal one another.

What does this mean?

Heath points out a long history of criticisms of Euclid's use of superposition to prove the congruence theorems.

Superposition Axiom

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Definition

An isometry is a bijection that preserves congruence of line segments.

Superposition Axiom:

If angle $BAC = DEF$ there is an isometry taking A to E and such that $B'A$ lies on DE and $C'A'$ lies on FE .

Consequences:

- 1 SAS
- 2 If $BA = DE$ there is an isometry taking A to E and B to D .

Solutions

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- 1 Spanish text from 50's: ignore the critique and really use superposition.
- 2 Hilbert: Assume only SAS

Three Frameworks

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- 1 Euclid
- 2 Hilbert
- 3 Birkhoff/Moise

Euclid

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Undefined Terms

points, lines, planes

Basic Relations

incidence, congruence,

Defined Relations

addition, multiplication

Axioms

(omitted continuity, 'sneaked in' superposition, no explicit congruence axioms)

Hilbert

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Undefined Terms

points, lines, planes

Basic Relations

betweenness, congruence

Defined Relations

addition, multiplication

Axioms

adds continuity, SAS

Birkhoff/Moise

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Undefined Terms

points, lines, planes, real numbers,

Basic Relations

length functions, angle measure functions, plus, times

Defined Relations

congruence (of segments, angles, figures)

Axioms

real number axioms; correspondence of geometry and numbers,
SAS

U.S. High School Curriculum

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The Birkhoff-Moise framework is almost universal.
One goal is to integrate algebra and geometry.
Another was to avoid the 'errors' of Euclid.

Difficulties with current curriculum

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- 1 Euclid's early propositions have real proofs; the basic facts of algebra are trivialities.

Difficulties with current curriculum

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- 1 Euclid's early propositions have real proofs; the basic facts of algebra are trivialities.

Difficulties with current curriculum

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- 1 Euclid's early propositions have real proofs; the basic facts of algebra are trivialities.
- 2 Problem: Students can't do (algebra) proofs.

Difficulties with current curriculum

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- 1 Euclid's early propositions have real proofs; the basic facts of algebra are trivialities.
- 2 Problem: Students can't do (algebra) proofs.
- 3 Solution: Take (geometry) proofs out of the curriculum.

Flattening out Geometry

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An 'honors' text in the U.S. has 24 postulates including:
SAS, SSS, ASA, HL,
3 (ruler, protractor, segment addition) tie geometry to unstated
axioms for real arithmetic

The role of Proof

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Proof is still a goal of state standards. But the textbooks are not adequate for students to learn how to prove.
There are many reasons; I focus on the mathematical one.

Diagnosis

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The fundamental problem is:
How do we come to grips with congruence and similarity?

Can one resurrect the principle of superposition?

Another Approach (Weinzweig/Hartshorne)

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Undefined Terms

points, lines, planes, rigid motions

Basic Relations

incidence, application of rigid motions

Defined Relations

congruence, addition, multiplication

Axioms

properties of rigid motions and basic geometry

Coming Events

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This talk is a summary of the course:

Math 592

Monday Nights 5-8

Fall 2007

A paper is available at

<http://www.math.uic.edu/~jbaldwin/pub/loggeomfor.pdf>

Lessons for Preparing Teachers

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The goals of proof are

- 1 not the mere verification of truth
- 2 but the gaining of understanding

Proof is a more efficient way retaining information than memorization.

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Euclid, Hilbert (google)

Hartshorne, Weinzweig

Solomonovich: review of modern books and introducing his

<http://www.solomonovich.com/geometry/textbook.html>

Raimi: Why the 'New Math' brought algebra into geometry

<http://www.math.rochester.edu/people/faculty/rarm/igno.html>