Math 503: Some open problems from FMT

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Some of the problems posed at Oberwolfach Feb. 1998.

1. (Vardi et al) Let B be a finite relational structure. Let C_B be the set of finite A that can be homomorphically embedded on A. Conjecture. C_B is either decidable in polynomial time or NP-complete (depending on the choice of B). (It is sufficient to consider directed graphs.)

(Vardi and Feder: see Vardi's homepage)

2. (Ash 1989) Fix a finite relational vocabulary τ. Consider the (finite) list of all equivalence classes of finite τ defined by equivalence by formulas of quantifier rank at most k. (This is NOT prefix rank). Let $S_{\tau,k}(n)$ be the number of such classes which are realized by a structure of size n. As a function of n for fixed k and τ is each such function eventually constant? (NP = co-NP implies the functions are not eventually constant.) Try to work out the functions for a single binary function and $k = 3$.

(C.J. Ash, A conjecture concerning the spectrum of a sentence, Math Logic Quarterly (1994) 393-397.)

3. (Invariant definability) (well-known problem) Fix finite similarity types ρ and σ and a sentence θ of type σ. (E.g. σ is a single binary relation and θ says S is one-one and onto except for a single first and last element. We call this *successor invariant*)

A class of finite ρ-structures in θ-invariantly definable if there is a $\rho \cup \sigma$-sentence ϕ such that the ρ structure A is in K if and only if every expansion of A to a model of θ satisfies ϕ.

Is there any class K of finite models which is successor invariantly definable and not first order as a class of ρ-structures?

If we replace 'successor' by order, the answer is yes.