Supplement to First Midterm: Math 215

February 24, 2007

Name:

If you missed items 3, 4, or 5 on the midterm, you should do the problem with the same number below. I will grade papers that are signed by two people indicating that you have convinced each other that the solutions are correct.

3 Prove from the axioms on ordered fields that
   1. There is no greatest real number a such that \( a^2 < 2 \).
   2. There is no greatest rational number a such that \( a^2 < 2 \).
   3. Explain the difference between the two results.

4 You may assume basic algebraic properties without explicit reference to the axioms in this problem. Recall that for integers \( d, s, n \), ‘d divides n’ means there is an s such that \( n = ds \). Let \( d, x \) and \( y \) be integers. Prove
   1. If \( d \) divides \( x \) and \( d \) divides \( y \) then \( d \) divides \( x + y \).
   2. If \( d \) divides \( x + y \) and \( d \) divides \( y \) then \( d \) divides \( x \).
   3. Explain the connection between the two results.

5 Consider the following three statements:
   1. If \( \frac{6x+5}{x+2} < 5 \), then \( x < 5 \).
   2. \( x < 5 \) then \( \frac{6x+5}{x+2} < 5 \).
   3. \( x < 5 \) if and only if \( \frac{6x+5}{x+2} < 5 \).

Which of them are true? Which one of them is needed to prove by induction that if \( a_{k+1} = \frac{6a_k+5}{a_k+2} \) then for all \( n, a_n < 5 \)?