**Math 215: Introduction to Advanced Mathematics**
Problem Set due April 24

1) Suppose $I$ is a countable set and that for each $i \in I$ we have a countable set $A_i$. Let $f_i : \mathbb{N} \to A_i$ be a surjection. Let

$$A = \bigcup_{i \in I} A_i = \{x : x \in A_i \text{ for some } i \in I\}.$$

Let $F : I \times \mathbb{N} \to A$ be the function $F(i, n) = f_i(n)$.

a) Prove that $F$ is a surjection.

b) Prove that $A$ is countable.

This excercise shows that a *countable union of countable sets is countable.*

2) a) Prove that the interval $(0, 1)$ is equipotent with the interval $(a, b)$. [Note: the interval $(c, d) = \{x \in \mathbb{R} : c < x < d\}$.]

b) Prove that the interval $(0, 1)$ is equipotent with the interval $(0, +\infty)$.

c) Prove that the interval $(0, +\infty)$ is equipotent with $\mathbb{R}$. Conclude that $(0, 1)$ is equipotent with $\mathbb{R}$.

3) Prove the square root of 10 is irrational.