1. Similar to Problem 10, page 116 but maybe slightly harder: If \((a, \infty) \subseteq (b, \infty)\) then \(a \geq b\).

To save typing I write \(A\) for \((a, \infty)\) and \(B\) for \((b, \infty)\). Note that \(A\) and \(B\) are sets of real numbers while \(a\) and \(b\) are real numbers.

I must show \(A \subseteq B\) implies \(a \geq b\). Now \(A \subseteq B\) means for all \(x\), if \(x \in A\) then \(x \in B\). By definition \(x \in A\) means \(x > a\) and \(x \in B\) means \(x > b\). So \(A \subseteq B\) translates to: for all \(x\), if \(x > a\) then \(x > b\). If, for contradiction it is not the case that \(a \geq b\), we would have \(b > a\). Applying our translation, we conclude, from the transitivity of the inequality relation that \(b > b\). This is impossible. So we conclude \(a \geq b\).

Remark. 1. The key point that involves the real numbers is the transitivity property of the ordering. There needs to be some reference to the properties of inequality to give a complete proof, although you might be less formal about it.

2. To say \(x\) is a value of \(A\) instead of \(x\) is an element of \(A\) or \(x \in A\) expresses the right idea but is not the conventional notation and will often be misunderstood.

2. The following problem was number 5 on the homework for Feb. 27.

Prove by truth tables the following two equivalences.

A. \((p \rightarrow q) \iff (\neg p \lor q)\).

(I.e. in English, \(( p \implies q)\) if and only if \(( \neg p) \lor q\).

B. \((\neg(p \rightarrow q) \iff (p \land \neg q)\).

(I.e. in English, \((\neg(p \implies q))\) if and only if \((p \land \neg q)\).

C. Use this information to write the negation of the proposition.

if \(f(a) < g(b)\) then \(a < b\).

(It originally said \(f(a) < g(a)\) – that introduces unnecessary confusion although it does not really change the problem. But I have rewritten it so it is easier to understand.)
Most of you got A and B correct but C wrong. Note that C asks you to write in a shorter form the meaning of: It is not the case that if $f(a) < g(b)$ then $a < b$.
HINT: Use part B.
3. Give an example of a set $A$ such that $A \not\subseteq P(A)$. 