

Notes on Third Essay

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1 Viewpoint

A crucial task in writing is to visualize the relationship between the author and the reader. This relationship determines the details and tone of the essay.

The written assignment framed the problem as explaining a mathematical/financial issue to a supervising engineer. I repeatedly said in class that the task was to write a memo.

Memo format: You may choose to write as a supervisor, to a supervisor, or to a group of colleagues. It is perfectly permissible to embed the memo in a larger context or just to write a memo. But make clear the position of the author and recipient of the memo. The relationship between the author and recipient indicated in the title will affect the details that are provided in the paper.

You may choose your own position and company.

It is completely inappropriate to include fragments of the assignment because they are directed to students, not other workers at the company.

What is a comptroller? Why should the comptroller want a very low cost?

The **title** of the paper should reflect its status as a memo. The title should not indicate that this is a class assignment.

If you elect the default assignment of writing a memo, the title will have the form:

To: XXXX,

From: YYYY

Subject: ZZZZ

XXXX and YYYY will be a name *and* a job title.

(A few of you have more inventive approaches that are fine.)

2 Earlier Essays

You may refer to your earlier essays; just list them as reference. If you do so, turn in the final version of Essay 2 so I can check the reference.

If you want to improve your score on earlier essays by rewriting certain arguments you can do this either in the body or an appendix of the current essay. If

you elect to do this you must a) make a note on the essay that you are doing so and b) include your final draft of Essay 2.

3 Conceptual problems

3.1 *Objects and Numbers*

One of the underlying themes of this course is the distinction between objects and numbers. This is a very particular example of the need in any writing to carefully distinguish different levels of abstraction. Thus, political parties are not people, continents are not countries, and fruit are not oranges.

In the context of these essays: ‘the deluxe set is *NOT* a harmonic series’. It can’t be. A set of blocks is not a sum of numbers. The correct statement asserts, ‘the height of the tower of blocks built using all blocks in the deluxe set is given by a harmonic series’.

But we can use the same mathematical expression to describe different physical situations. Thus, the mathematical expression $\sum_0^\infty \frac{1}{2^n}$ can be used in one context to represent the areas of successively smaller squares (picture proof) and in another to represent the edge length of successively smaller cubes (height of standard set). The great utility of mathematics stems from the ability to manipulate these formulas in the abstract and apply the results to specific cases. The difficulty in writing these essays is to distinguish between the physical structures and their mathematical descriptions.

Be sure you understand the difference between area, surface area, and volume. Use appropriate units: square feet for area and surface area, cubic feet for volume.

3.2 *The main explanation*

I list below the key steps in explaining the estimate of the cost to paint the deluxe set. They don’t have to appear in the order I give here.

We want to evaluate $\sum_0^\infty \frac{1}{n^2}$. Explain in a couple of sentences the fact that for any m

$$\sum_0^\infty \frac{1}{n^2} < \sum_0^m \frac{1}{n^2} + \int_m^\infty \frac{1}{x^2} dx.$$

This explanation just invokes the interpretation of integral as area under a curve. But you must be very specific about which curve and on which interval on the x -axis.

Explain how to evaluate the integral.

Connect this mathematical representation with the problem of computing the sum of the areas of one side of each cube in the deluxe set.

Connect the estimate of the sum of the areas of one side of each cube in the deluxe set with the problem of computing the sum of the surface areas of each cube in the deluxe set.

Connect the estimate of the sum of the surface areas of each cube in the deluxe set with the price of painting the deluxe set.

3.3 *Volume*

The volume of the deluxe set should be shown to be less than $1 \frac{1}{4}$ cubic feet by using the same method as this estimate of surface area.

3.4 *The role of Euler's theorem*

The main calculation should use interaction between infinite series and integrals explained in 'The Mathematics of Essay II' and summarised in the previous subsection. You may refer to the fact that Euler provided a more exact solution. But this should be for 'culture'. The main explanation of the essay should be understandable by a C student in Math 181.

Note that while we could solve the surface area problem by quoting Euler, we don't have a similar appeal to authority to compute the volume. But, we can use the 'integral' method we used to estimate the surface area.

3.5 *Surface Area*

Many students have described incorrectly the relationship between the surface area of a $1 \times 1 \times 2$ box and the sum of the surface areas of the cubes in the deluxe set. Note that in Essay 2, we estimated the surface area as less than 12 square feet even though surface area of the box is 10 square feet.

The assignment said that a 'colleague' had estimated the surface area as less than 10.2 square feet. You must explain his reasoning and improve his result. He is proposing that the cost is \$102 and you must show it is less than \$100.

3.6 *lower bounds*

To show for example that the cost is at least \$90, list enough blocks so that painting them will cost \$90. It is a small finite number.

Similarly, list a few blocks that cannot be put in Joe's Messerschmidt and say why.

Hint: This is really, really easy; if you are having trouble you are working too hard. I said in the assignment that it should convince a 6th grader. A first grader can see why the blocks won't fit in the Messerschmidt. I said sixth grader because the paint problem uses the word 'surface area'.

Even if you have answered Joe's question in this way, I want you to explain why the volume of the deluxe set is less than $1 \frac{1}{4}$ cubic feet. (Show this using the same technique as finding the upper bound on surface area.) Then explain this apparent paradox (a volume of less than $1 \frac{1}{4}$ cubic feet will not fit in a box that holds $1 \frac{1}{4}$ cubic feet).

4 Some notes on the comments

I repeat from the first set of comments some further explanation for the abbreviations in the corrections.

Use fewer words.

INC Write complete sentences. There were several causes of incomplete sentences. Often they arose as run-on sentences. They also arose because there was no major verb in the sentence. Rather, two relative clauses were awkwardly spliced together.

RED Redundant: Repeating the same idea twice is row doesn't explain it. Make sure your 'explanation' really adds information.

TMW Two many words: Write more concisely and directly. Say what you mean to say rather than explaining that you would like to say it.

5 Dicta on Style

PROOFREAD

Proofread! Proofread! Proofread!

Make sure your assignment is done by Sunday April 26. Then read it over on Tuesday and correct errors; they will exist.

Infinitives are often more direct than participles. Write 'aim to minimize' instead of 'are interested in minimizing'.

Employ words, especially verbs, that have meaning. Avoid colorless verbs such as 'is' and 'used'.

Don't confuse 'then' and 'than'.

Check that subject and verb agree in number.

Make sure that each sentence has a subject and main verb. Be careful that you have not mistaken the verb in a relative clause for a main verb in a sentence.

6 Semicolons

Many students have regressed in the use of semicolons; do not use a coordinating conjunction such as 'and' immediately after a semicolon. Review the usage of semicolons. For example, consult:

<http://owl.english.purdue.edu/owl/resource/607/04/>