Math 300, Spring 2009, Essay 3

Improper Integrals and sums of series

The audience for this essay is someone like yourself, who has taken calculus and knows the basic notions but may have forgotten the details of exotic topics like improper integrals. The difficulty is one that may often be faced in business. How do you explain to a supervising engineer who studied calculus 30 years ago that the particular solution you are proposing for a problem should be tried?

Again, we are dealing with the products of the Blocks Unlimited Store but various customers are asking trickier questions.

The Ozimandias Corporation want to paint their blocks with a platinum gild, which is extremely expensive. Naturally, they want to minimize the amount of paint needed. One of your colleagues asserts that one needs only enough paint to cover not 12 square feet but 10.2 square feet. Can the paint cost of this fancy set be reduced to under \$ 100 per set if it costs \$10 to cover one square foot with platinum gild?

You also must deal with the comptroller of Ozimandias Corporation who would like to lower the price even further. Give a convincing argument that the price cannot go below \$90 a set. (This should convince someone who knows no calculus or algebra, even though the comptroller does; he is being difficult. Indeed, it should convince a 6th grader.)

An aging motorcyclist, Joe Ventura, bought an antique Messerschmitt automobile. Google or look on home page for link to a picture. He wants to fit the Deluxe set in a space which is $1 \times 1 \times 1\frac{1}{4}$ feet. Can he do it? (There are two problems here. Is the volume of the Deluxe set less than $1\frac{1}{4}$ cubic feet? Can the blocks actually be packed in the specific shape which fits in the back of a Messerschmidt?)

The essay should confront several issues. How does the relationship between the area under a curve and the integral explain the approximation formulas for series.? How do you relate the surface areas of the blocks to the integral $\int_1^\infty \frac{1}{n^2}\,dn$ or the volumes of the blocks to the integral $\int_1^\infty \frac{1}{n^3}\,dn$? What justifies the specific cost and packing estimates for this problem?