Definition 1: A pregeometry is a set \( G \) together with a dependence relation 
\[ \text{cl} : \mathcal{P}(G) \to \mathcal{P}(G) \]
satisfying the following axioms.

A1. \( \text{cl}(X) = \bigcup \{ \text{cl}(X') : X' \subseteq \text{fin} X \} \)

A2. \( X \subseteq \text{cl}(X) \)

A3. \( \text{cl}(\text{cl}(X)) = \text{cl}(X) \)

A4. If \( a \in \text{cl}(Xb) \) and \( a \notin \text{cl}(X) \), then \( b \in \text{cl}(Xa) \).

If points are closed the structure is called a geometry.

Definition 2: A geometry is homogeneous if for any closed \( X \subseteq G \) and \( a, b \in G - X \) there is a permutation of \( G \) which preserves the closure relation (i.e. an automorphism of the geometry) which fixes \( X \) pointwise and takes \( a \) to \( b \).

Exercise 3: If \( G \) is a homogeneous geometry, \( X, Y \) are maximally independent subsets of \( G \), there is an automorphism of \( G \) taking \( X \) to \( Y \).

Definition 4: 1. The structure \( M \) is strongly minimal if every first order definable subset of any elementary extension \( M' \) of \( M \) is finite or cofinite.

2. The theory \( T \) is strongly minimal if it is the theory of a strongly minimal structure.

3. \( a \in \text{acl}(X) \) if there is a first order formula with finitely solutions over \( X \) which is satisfied by \( a \).

Definition 5: Let \( X, Y \) be subsets of a structure \( M \). An elementary isomorphism from \( X \) to \( Y \) is 1-1 map from \( X \) onto \( Y \) such that for every first order formula \( \phi(v) \), \( M \models \phi(x) \) if and only if \( M \models \phi(fx) \).

Exercise 6: Find \( X, Y \) subsets of a structure \( M \) such that \( X \) and \( Y \) are isomorphic but not elementarily isomorphic.

Exercise 7: Let \( X, Y \) be subsets of a structure \( M \). If \( f \) takes \( X \) to \( Y \) is an elementary isomorphism, \( f \) extends to an elementary isomorphism from \( \text{acl}(X) \) to \( \text{acl}(Y) \).
Exercise 8 Show a complete theory \( T \) is strongly minimal if and only if it has infinite models and

1. algebraic closure induces a pregeometry on models of \( T \);
2. any bijection between acl-bases for models of \( T \) extends to an isomorphism of the models.

Exercise 9 A strongly minimal theory is categorical in any uncountable cardinality.