Now we prove ‘Morley’s method’ for Galois types.

**Lemma 1** [II.1.5 of 394] If $M_0 \leq M$ and $M$ is huge we can find an EM-set $\Phi$ such that the following hold.

1. The $\tau$-reduct of the Skolem closure of the empty set is $M_0$.
2. For every $I$, $M_0 \leq EM(I, \Phi)$.
3. If $I$ is finite, $EM_\tau(I, \Phi)$ can be embedded in $M$.
4. $EM_\tau(I, \Phi)$ omits every galois type over $N$ which is omitted in $M$.

Proof. Let $\tau_1$ be the Skolem language given by the presentation theorem and consider $M$ as the reduct of $\tau_1$-structure $M^1_1$. Add constants for $M_0$ to form $\tau_1'$. Now apply Lemma ?? to find an EM-diagram $\Phi$ (in $\tau_1'$) with all $\tau$-types of finite subsets of the indiscernible sequence realized in $M$. Now 1) and 2) are immediate. 3) is easy (using clause 5) of Theorem ?? since we chose $\Phi$ so all finite subsets of the indiscernible set (and so their Skolem closures) are realized in $M$.

The omission of Galois types is more tricky. Consider both $M$ and $N = EM_\tau(I, \Phi)$ embedded in $M$. Let $N_1^1$ denote the $\tau_1'$-structure $EM(I, \Phi)$. We need to show that if $a \in N$, $p = ga - tp(a/M_0)$ is realized in $M$. For some $e \in I$, $a$ is in the $\tau_1$-Skolem hull $N_e$ of $e$. (Recall the notation from the presentation theorem.) By 3) there is an embedding $\alpha$ of $N_e$ into $M^1$ over $M_0$. $\alpha$ is also an isomorphism of $N_e \upharpoonright \tau$ into $M$. Now, by the model homogeneity, $\alpha$ extends to an automorphism of $M$ fixing $M_0$ and $\alpha(a) \in M$ realizes $p$. □

This has immediate applications in the direction of transferring categoricity.

**Theorem 2** Suppose $M \in K$ omits a Galois type $p$ over a submodel $M_0$ with $|M| \geq \mu(|M_0|)$. Then there is no regular cardinal $\lambda \geq |M|$ in which $K$ is categorical.

Proof. By Lemma 1, there is a model $N \in K$ with cardinality $\lambda$ which omits $p$. But by Lemma ??, the unique model of power $\lambda$ is saturated. □

In [?] Shelah asserts the following result:

**Theorem 3** If $K$ is categorical in a regular cardinal $\lambda$ and $\lambda > \mu(|\tau|)$ then $K$ is categorical in every $\theta$ with $\mu(|\tau|) \leq \theta \leq \lambda$.
Here is a sketch of the argument. We have shown that there are saturated models of power $\theta$ for every $\theta < \lambda$. The obstacle to deducing downward categoricity is that Theorem 1 only allows us to transfer the omission of types when the model omitting the type is much bigger than the domain of the type. The first step in remediying this problem is to show that all types are determined by ‘relatively small’ subtypes. More precisely, we need the notion that Grossberg and Van Dieren [?] have called $\chi$-tame and Shelah [?] refers to has ‘having $\chi$-character’. We add an extra parameter to be careful.

**Definition 4** We say $K$ is $(\chi, \mu)$-tame if for any saturated $N \in K$ with $|N| = \mu < \lambda$ if $p, q, \in ga - S(N)$ and for every $N_0 \leq N$ with $|N_0| \leq \chi$, $p \restriction N_0 = q \restriction N_0$ then $q = p$.

Shelah asserts the following in Sections II.1 and II.2.3 of the published version of [?]. The published proof is incomplete; I haven’t yet seen the corrections. But it seems to use only Ehrenfeucht-Mostowski type methods.

**Theorem 5** Suppose $K$ is $\lambda$-categorical for $\lambda \geq \mu(\tau)$ and $\lambda$ is regular. Then $K$ is $(\chi, \chi_1)$-tame for some $\chi$ and any $\chi_1$ with $\chi < \mu(\tau) \leq \chi_1 \leq \lambda$.

The naive argument would give $\chi = \mu(\tau)$ since one is omitting types. But omitting in every cardinal below $\mu(\tau)$ is as good as in $\mu(\tau)$ so the conclusion becomes for some $\chi$ with $\chi < \mu(\tau)$.