Coordinatization

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Context

We are working in the situation with Hilbert's axiom groups I, II, III (incidence, order, and congruence) and the parallel postulate.

We have proved SAS, ASA, and ASA.

Order

Fix any line ℓ and two points 0, 1 on that line. We define a linear order on the points on that line.

We say a is positive if a is between 0 and 1 or 1 is between 0 and a. If a is neither 0 nor positive a is negative. Now $a < b$ if

1. b is positive and a is not.
2. a and b are positive and a is between 0 and b.
3. a and b are negative and b is between 0 and a.

Connection to Hilbert

Hilbert's objects were equivalence classes of segments (under congruence. We change two things.

1. We fix a representative $(0, a)$ of the equivalence class.
2. But we also encode direction since we split the class into two - $(0, a)$ and $(-a, 0)$.

Note $-a$ is only expository- I haven't defined it; although I could.

Addition

Let $a, b \in \ell$. Then $a + b = c$ if

1. a, b, c are positive, $a < c$, and the interval (a, c) is congruent to $(0, b)$.
2. a is positive and b is negative and $c < a$ such that (c, a) is congruent to $(b, 0)$.
3. EXERCISE: b is positive and a is negative.

Multiplication

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The definition of multiplication by proportions seems to be definitely about positive numbers.

We can extend to all numbers by defining $-a$ and $|a|$ and then extend multiplication to negative number by the usual rules: e.g. $a < 0, b > 0$ implies $ab = (-a)b$.

But the absence of a geometric model may explain why students have so much trouble.