Assignment 8 due 0ct 29

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Feel free to e-mail with questions.

A) A real number a is rational if there are integers m and n such that $\frac{m}{n} = a$.

Show the square root of 2 is irrational. (How many times have you a) seen b) done this proof before?)

Write a sentence or two explaining the connection between this assignment and the one last week to prove there are two segments that are incommensurable.

B)

1. Given magnitudes a, b, construct three segments so that the length of one is the sum of a and b.
2. Given magnitudes a, b, construct three segments so that the length of one is the product of a and b.

Reflection Questions

1. What is the connection between 'magnitude' and 'segment'?
2. What argument can we give that our notion of 'sum' is a good one?
3. What argument can we give that our notion of 'product' is a good one?

If you get far enough into showing the 'notion of product is good' you will need the theorem of Pappus to show multiplication is commutative. I hope you will at least get far enough to see that something like this is necessary.

Theorem of Pappus (Affine version)

Suppose two lines cross and A,B,C are on one, A', B', C' on the other with none of the named points at the intersection and each triple on a single ray.

If $CB' \| BC'$ and $AB' \| A'B$ then $CA' \| AC'$.

Hilbert call this Pascal's theorem but no one else does.

C. Prove the following three propositions are equivalent.

1. corresponding angles are equal, and

2. alternate interior angles are equal, and
3. the sum of two interior angles on the same side equal to two right angles

You may assume that 'all straight angles are equal'. I don't intend to assign busy work so an acceptable answer to problem C is, 'I could do this in my sleep.'

D. Read the article by Raimi, listed on the web page as:Raimi: Why the New Math brought algebra into geometry.

This will be helpful background on why problems A and B have been assigned as well as some hints to solving B.