## Superposition

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## Congruence and Similarity

1 Another take on Congruence
2 Similarity -the role of arithmetic and geometry

## Congruence of triangles-Hilbert

Basic notions: point, line, incidence, between, congruence (angle,segment)
defined terms: segment angle
Axioms included SAS

## Congruence of triangles-tranformation approach

A transformation of a geometry is a permutation of the points and a permutation of the lines that preserves incidence.
If it also preserves congruence of segments and angles then it is
a rigid motion
Now a key axiom is:

## Superposition Axiom:

If angle $B A C=D E F$ there is an isometry taking $A$ to $E$ and such that $B^{\prime} A^{\prime}$ (i.e. $B E$ ) lies on $D E$ and $C^{\prime} A^{\prime}$ lies on $F E$.
yields SAS

## Exercises

Exercise: Prove SAS from this axiom and Hilbert order and congruence up to but not including SAS.
Exercise: Prove that if $A B \cong C D$ there is rigid motion mapping $A$ to $C$ and $B$ to $D$.

## Congruence-Weinzweig

Basic notions: point, line, incidence, between, motion defined terms: segment angle, congruence Definition. Two figures are congruent if they are mapped to each other by a motion.

Axioms guarantee that the group of motions are 'the right group':
SAS is a theorem.

