(1) Prove or provide a counterexample for each of the following:
   (a) If $m$ divides $ab$ then $m$ divides $a$ or $m$ divides $b$.
   (b) If a prime $p$ divides $ab$ then $p$ divides $a$ or $m$ divides $b$.

(2) Solve the recursion relation
   $$a_n = 6a_{n-1} - 9a_{n-2}$$
   with initial conditions $a_0 = 1$ and $a_1 = 3$.

(3) Use mathematical induction to prove the following identity for all natural
numbers $n$.
   $$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

(4) If $x^k + 1$ is prime then $k$ is a power of two.

(5) Find a 1-1 correspondence between $\{\langle a, b \rangle : a, b \in \mathbb{R}\}$ and the complex
numbers, i.e., $\{a + bi : a, b \in \mathbb{R}\}$. What does this say about the relation
between the cardinality of the set of reals $\mathbb{R}$ and the cardinality of the set
of complex numbers.