

FROM 1.1:

1. (b) This is true only if *both* statements connected by the *and* are true. The statement  $(4 \neq 2 + 2)$  is false, so the whole statement is false.

(i) This statement is true unless the first statement is true and the second statement is false. This is in fact the case, so the implication is false.

3. (b) Either  $x$  is not a real number or  $x^2 + 1 \neq 0$ .

(f)  $(ab)c = a(bc)$  for all  $a$ ,  $b$ , and  $c$ .

(h)

$$(\forall x)(\forall y)[x > 0 \rightarrow x^2 + y^2 > 0]$$

That is,

$$(\forall x)(\forall y)[\neg(x > 0) \vee (x^2 + y^2 > 0)].$$

Take the negation:

$$\neg[(\forall x)(\forall y)[\neg(x > 0) \vee (x^2 + y^2 > 0)]].$$

is equivalent to

$$[(\exists x)(\exists y)\neg[\neg(x > 0) \vee (x^2 + y^2 > 0)]]$$

which by DeMorgan's law is equivalent to

$$[(\exists x)(\exists y)[(x > 0) \wedge \neg(x^2 + y^2 > 0)]]$$

which is equivalent to

$$[(\exists x)(\exists y)[(x > 0) \wedge (x^2 + y^2 \leq 0)].$$

4. (d) Converse:  $(a = 0 \text{ or } b = 0) \rightarrow ab = 0$ .

Contrapositive:  $\neg(a = 0 \text{ or } b = 0) \rightarrow \neg(ab = 0)$ . This is the same as  $(a = 0 \text{ and } b = 0) \rightarrow ab \neq 0$ .

5. (b) For all real  $x$ ,  $2^x$  is nonnegative.

FROM 1.2:

3. Converse: " $x + 2$  is an even integer implies  $x$  is an even integer." This is true. Suppose that the hypothesis is true, that is, assume that Then there is some integer  $y$  such that  $x + 2 = 2y$ . Then  $x = 2y - 2 = 2(y - 1)$  so  $x$  is divisible by 2 and therefore even.

4. A statement  $p \leftrightarrow q$  is true if  $p \rightarrow q$  and  $q \rightarrow p$  are both true. Let  $p$  be the statement ' $x$  is an even integer' and  $q$  be the statement " $x + 2$  is an even In problem 2 we saw that  $p \rightarrow q$  is true and in problem 3 we saw that  $q \rightarrow p$  is true, so the statement " $x$  is an even integer  $\leftrightarrow x + 2$  is an even integer" is

6. No. For the implication  $p \rightarrow q$  to be false,  $p$  must be true and  $q$  must be false. If both of these hold, the implication  $q \rightarrow p$  must be true since its hypothesis is false.