1.4

2.c). Simplify \[ (p \to q) \lor (q \to r) \land (r \to s). \]

\[ (((\neg p) \lor q) \lor ((\neg q) \lor r)) \land (r \to s) \]

\[ ((\neg p) \lor q) \lor ((\neg q) \lor r) \land (r \to s) \]

\[ ((\neg p) \lor r) \land (r \to s) \]

\[ 1 \land (r \to s) \]

Assume that the argument is not valid. This means that we can find truth values for \( p \), \( q \), \( r \), and \( s \) such that the premises are true but the conclusion is false. Since \( s \to (r \lor q) \) is false, we must have \( s \) true and \( r \lor q \) false. But this means both \( r \) and \( q \) are false. Since \( p \to q \) is true and \( q \) is false, \( p \) must be false. But then \( q \lor (\neg r) \) is true and \( p \land s \) is false, contradicting the truth of \( (q \lor (\neg r)) \to (p \land s) \). Hence we have a contradiction, so the argument is valid.

5. c) Let \( p = \) “I stay up late at night." and \( q = \) “I am tired in the morning.” Then the given argument is

\[ p \to q \]

\[ 
eg q \]

\[ \neg p \]

which is valid by modus tollens.

1.5

1.d)

\[ p \to q \]

\[ (q \lor (\neg r)) \to (p \land s) \]

\[ s \to (r \lor q) \]

Assume that the argument is not valid. This means that we can find truth values for \( p \), \( q \), \( r \), and \( s \) such that the premises are true but the conclusion is false. Since \( s \to (r \lor q) \) is false, we must have \( s \) true and \( r \lor q \) false. But this means both \( r \) and \( q \) are false. Since \( p \to q \) is true and \( q \) is false, \( p \) must be false. But then \( q \lor (\neg r) \) is true and \( p \land s \) is false, contradicting the truth of \( (q \lor (\neg r)) \to (p \land s) \). Hence we have a contradiction, so the argument is valid.

5 c) Let \( p = \) “I stay up late at night.”, and \( q = \) “I am tired in the morning.” Then the given argument is

\[ p \to q \]

\[ 
eg q \]

\[ \neg p \]

which is valid by modus tollens.

d) The given argument is

\[ p \to q \]

\[ 
eg p \]

\[ \neg q \]

This is not valid, since when \( p = F \) and \( q = T \), the hypotheses are true, but the conclusion \( \neg q = F \) is false.