1. Solve the recursion relation
\[ a_n = 6a_{n-1} - 9a_{n-2} \]
with initial conditions \( a_0 = 1 \) and \( a_1 = 3 \).

Solution:
Applying the algorithm: solve \( x^2 - 6 + 9 = 0 \) which has double roots at \( x = 3 \).
So the solution has the form
\[ a_n = c_1(3^n) + c_2n(3^n). \]
So we try to solve the system of equations:
\[ 1 = a_0 = c_1(3^0) + c_20(3^0) \]
\[ 3 = a_1 = c_1(3^1) + c_23(3^1) \]
and we get \( c_1 = 1 \) (Don’t forget to substitute 0 for the \( n \) in \( c_2n(3^n) \).) and \( 3 = 3c_1 + 3c_2 \) so \( c_2 = 0 \).
The solution is \( a_n = 3^n \).

4. If \( 2^k + 1 \) is prime then \( k \) is a power of two.
We show the contrapositive by showing that if \( k \) is not a power of two then \( x^k + 1 \) factors:
If \( k \) is odd the factorizations is \( x^k + 1 = (x + 1)(x^{k-1} - x^{k-2} + \ldots 1) \).
If \( k \) is twice an odd number, the factorizations is
\[ x^k + 1 = (x^2 + 1)(x^{k-2} - x^{k-4} + \ldots 1). \]
Thus, if \( k \) is not a power of 2 (i.e if \( k \) has an odd factor), either \( 2 + 1 = 3 \) or \( 2^2 + 1 = 5 \) is a proper factor of \( 2^k + 1 \) and \( 2^k + 1 \) is not prime.