Midterm Exam: MTHT 400
Methods of Teaching Secondary Mathematics
Summary of grading

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In the following I indicate the maximum number of points given for each part of each problem and give or describe a correct answer.

The instructions were:

Do 4 of the 5 problems. In all problems be careful to label the variables and write units.

1. *A high school class conducts the following experiment. They weigh a number of pieces of fruit (some grapes, apples, grapefruit, and pumpkins); they also measure the circumference of each piece of fruit. After making a table of circumference versus weight, they use the cubic regression program on the calculator to give them a cubic function for weight in terms of circumference.*

Why is it reasonable to expect this relationship to be cubic? What is a plausible interpretation of the coefficient of the cubic term? Give arguments for or against this being a good example of a mathematical model.

Solution The fruit involved are all roughly spherical. So the volume is proportional to the cube of the radius (and the radius is linear in the circumference) so the volume is cubic in the circumference. (10 pts) (Some noticed that in general volume is cubic in a linear dimension). Moreover, if we assume the fruit is homogeneous the weight should be proportional to the volume (5 pts) so the weight is cubic in the circumference and the coefficient of the cubic term is a constant times the density of the fruit (5 points). Some drawbacks to this analysis are that the density can vary by the kind (or size) of fruit and that fruit (especially pumpkins) are not very homogeneous (5 points).

2. *A student writes:* $(2x+1)(3x^2+2x+2) = 6x^3+4x+3x^2+2$ *and explains 'I foiled it'. How would you help this student understand how to perform this calculation. What is the key property of multiplication involved? Suggest a strategy to help a 'visual learner'.*

Solution The student made the mistake of applying the foil acronym which only works for two binomials to a product of a binomial and trinomial (7 points).

Explain how the *distributive* law leads to a complete solution (8 points). The expected explanation for a 'visual learner' (7 points) was the use of an area model; some people used a nice substitution of capital A, B, C for one of the polynomials to show the distribution.

3. *a) (15 points) A ring is an integral domain if $ab = 0$ implies $a = 0$ or $b = 0$. Explain how the fact that the real numbers are an integral domain justifies the usual method of solving a quadratic equation by factoring.*

Solution: Suppose we have factored $ax^2 + bx + c$ as $(a_1x + r_1)(a_2x + r_2)$. Suppose g is a solution. Then,

$$(a_1g + r_1)(a_2g + r_2) = 0$$

and $(a_1g + r_1)$ and $(a_2g + r_2)$ are real numbers. Since their product is zero, the given definition of integral domain shows that either $a_1g + r_1 = 0$ or $a_2g + r_2 = 0$. So g must be $-r_1/a_1$ or $-r_2/a_2$. (Words to this effect get a score of 15.) But, simple substitution shows that both of these are solutions so we have found all the roots.

b) (10 points) A certain algebra text list the basic properties of the real numbers by giving the usual field axioms; Addition and multiplication are both associative and commutative; there is an additive identity 0 and a multiplicative identity 1; every number has an additive additive inverse; every non-zero number has a multiplicative inverse; multiplication distributes over addition. Why is a field an integral domain?

Solution: Suppose $a \neq 0, b \neq 0$ and $ab = 0$. Multiply by a^{-1} to get $b = 0$; contradiction.

4) How would you explain to beginning algebra students that

$$\frac{1}{a+b} \neq \frac{1}{a} + \frac{1}{b}?$$

Which of the two expressions is bigger? (Do you have to worry about sign in answering this question?)

Solution: Give a numeric example to show the inequality (10 points). Carry out the addition of the fractions. (I gave this 10 points but it should be very much the second thing you do with a student if at all.) Finally, the truth of the inequality (5 points) very much depends on the sign of a, b. Do some examples.

5) *Two cities, Moose Jaw and Bear Skull, are 100 miles apart. Going at a constant speed, suppose car A makes the trip from Moose Jaw to Bear Skull in time t_A. Starting at the same time and also going at constant speed, car B makes the trip from Bear Skull to Moose Jaw in time t_B.*

Find the elapsed time until they pass each other as a function of t_A and t_B; explain why the distance between the cities does not affect this answer.

Solution: The speed of A is $\frac{100}{t_A}$, the speed of B is $\frac{100}{t_b}$. Let t be the time until they meet. Then

$$\frac{100}{t_A} \cdot t = 100 - \frac{100}{t_B} \cdot t$$

Divide by 100; add $\frac{1}{t_B} \cdot t$ to both sides of the equation to get:

$$\frac{1}{t_A} \cdot t + \frac{1}{t_B} \cdot t = 1.$$

That is,

$$t(\frac{1}{t_A} + \frac{1}{t_B}) = 1$$

or

$$t = \frac{t_A + t_B}{t_A t_B}.$$

(This got 20 points; 10 points plus partial credit for those who found expression for the distance travelled until they meet and then failed in a second step to find the time of meeting.)

(Maximum of 5 points for explaining that the answer did not depend on the distance the vehicles travelled because 'distance *divided out*' or depends only on the speeds of the two vehicles.)