Homework: MTHT 400
Rubric for Logic Problems
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The definitions of converse and contrapositive can easily be found on the net. (E.g. on Dr. Math
http://mathforum.org/library/drmath/view/54198.html) where there are a number of examples explaining the notions asked about below.

For negation see for example
http://en.wikipedia.org/wiki/Negation
For symmetric, reflexive etc, see for example
http://mathworld.wolfram.com/SymmetricRelation.html

grading: 1a,1b,1c were worth one point for converse, one for contrapositive, similarly 1 point for whether each of converse, contrapositive is true. Plus a bonus point for those who observed that an implication is equivalent to its contrapositive. 4a, 4b one point each 5,6 two points, 7 points. Since most people were unable to find the assigned reading I didn't record a grade on the responses.

1. Write down the converse and contrapositive of each of the following implications.

   The converse of an implication is *never* **equivalent to the original implication. Of course, it can happen that both are true**

   (a) If a and b are integers then ab is an integer.
   converse: If ab is an integer then a and b are integers.
   contrapositive: If ab is not an integer then a is not an integer or b is not an integer.

   (b) If x is an even integer then x^2 is an even integer. **converse:** If x^2 is an even integer then x is an even integer.
   contrapositive: If x^2 is not an even integer then x is an not even integer.

(c) Every planar graph can be colored with at most 4 colors.

Reword as: If a graph is planar then it can be colored with at most 4 colors. **converse:** If a graph can be colored with at most 4 colors then it is planar.

contrapositive: If a graph can not be colored with at most 4 colors then it is not planar.

2. For each of the previous examples is the converse true? Why?

   There are easy counterexamples to 1a) and 1b) if you assume x can be rational number or in the case of 1b) an irrational number. If you assume the statement was about integers; then the converse of 1b) is easily proved.

3. For each of the previous examples is the contrapositive true? Why?

   Yes, because the contrapositive of a true is implication is true.

4. Write down the negation of each of the following statements in clear and concise English.

   (a) Either x is not a real number or $x > 4$.
   x is a real number and $x \geq 4$.

   (b) There exists a real number x such that $n > x$ for every integer n.
   For every real number x there is a natural number n with $n \leq x$.
   Or, there is no real number x with $n > x$ for every integer n.

5. Let n be an integer; prove that n is odd if and only if n^3 is odd.

   A ninth grade exercise shows $(2k+1)^3$ is odd.

6. What is wrong with the following argument which purports to prove that every relation which is symmetric and transitive must necessarily be reflexive as well.

   Suppose R is a symmetric and transitive binary relation on a set A and let $a \in A$. Then for any b with $\langle a, b\rangle \in R$, we have also $\langle b, a\rangle \in R$ by symmetry. Since we now have both $\langle a, b\rangle \in R$ and $\langle b, a\rangle \in R$, we have $\langle a, a\rangle \in R$ as well, by transitivity. Thus $\langle b, a\rangle \in R$ and R is reflexive.

   Actual Error: There is nothing guaranteeing that for an arbitrary $a \in A$, there is $b \in A$ with $\langle a, b\rangle \in R$. One student gave a complete response by exhibiting a counterexample.

   Frequent mistake by MTHT 400 students They assert that this is not failure of transitivity because transitive (aRb and bRc implies aRc) only holds when a, b, c are distinct. This is a fundamental misunderstanding of the use of variables.

7. You say, ‘every square is a rectangle’ and a student looks at you with complete incomprehension. What can you do? What might the student be failing to understand?

   Most students replied with reasonable explanations of the distinction between squares and rectangles. This was worth two points. Only a few guessed what I really was after. The student may not understand what a phrase like ‘all squares are rectangles means’. One should give examples like ‘all lollipops are candy but not all candies are lollipops’ or ‘all dogs are animals but not all animals are dogs’.

8. Read the article VOCABULARY CONCERNS IN THE MASTERY OF MATHEMATICS: COLLEGE ALGEBRA ...

   www.math-cs.cmsu.edu/ mjms/2003.1/Francis00.ps

   Are some of his examples ‘distinctions without a difference’. I think at least two of the examples are terms for which you can find large numbers of mathematicians who adopt either of two conventions. (A similar example is: are the natural numbers $0, 1, 2, 3 \ldots$ or $1, 2, 3, \ldots$?). Can you think of other examples that could have been used in the pretest. See if you can locate the controversial examples. We will discuss these issues on November 16.