Order of Growth

This assignment has two purposes: understanding the notion of order of growth as a way to organize the study of functions; considering some advantages and disadvantages of calculators.

1. Explain in words for a 12th grade the meaning of: the function f is bigger than the function g. Give some examples. (You will have to choose a specific meaning for 'bigger'.)

2. One meaning for 'bigger' is: f has higher order of growth that g if

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = +\infty.$$

We write $g \prec f$.

(You may have seen some similar notions concerning big oh and little oh. We will just use this definition of 'bigger' here.)

Make up some examples to illustrate this concept with a calculator or a computer program like maple. Sketch the result and hand it in. What are two different functions that have the same order of growth?

3. Let $g(x) = x^{10}$ and $f(x) = 10^x$. Using a calculator or maple guess which of these two has higher order of growth. Explain your answer and discuss how you would have high school seniors (precalc) study this problem.

Fact 1. Recall L'hôpital's rule: Suppose $f(x)$ and $g(x)$ are differentiable real functions with $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} g(x) = +\infty$. If $\frac{\lim_{x \to \infty} f'(x)}{\lim_{x \to \infty} g'(x)}$ exists then $\frac{\lim_{x \to \infty} f'(x)}{\lim_{x \to \infty} g'(x)} = \frac{\lim_{x \to \infty} f(x)}{\lim_{x \to \infty} g(x)}$.

4. Use L'hôpital's rule to decide which of $g(x) = x^{10}$ and $f(x) = 10^x$ has the higher order of growth. (This will be a little easier to do if you use e^x instead of 10^x; if you are having trouble you might do that question first.) Return to your calculator picture and explain what is going on.

Fact 2. Recall that the graph of the inverse function of f is obtained by reflecting the graph of f through the line $y = x$.

5. Use Fact 2 about the inverse function to deduce from the result of problem 4, that $\log_{10}(x) \prec x^{1/10}$. This argument should be given as a sequence of picture that you can draw on the board. There should be no new calculating of limits in your answer to question 5; just sketch graphs.

6. Four important classes of function are polynomial, logarithmic, exponential, and linear. Explain the relationship of these classes in terms of order of growth. For each class, give a natural concrete example that you would use in 11th grade of a real problem modeled by a function in that class.