rubric: Mersenne Primes

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The papers are scored 5 points for each of the explicit problems now given below and 5 points for discussions and examples of Mersenne primes. Since only one person conjectured and solved the two explicit problems listed below, everyone else will have a week to solve them (for a maximum of 4 points each) and then I will grade that part and the rest of the paper.

We continue the saga of trying to figure out to assign exploratory problems in a way that allows students to explore but still leads to specific problems. The assignment asked ‘try to give a property that guarantees that $2^j - 1$ (later $2^j + 1$) is prime or composite. As most of you discovered, for prime this is open and very important. As some of you discovered, the problem of giving sufficient conditions for $b$ to be composite is doable by high school students. Let me rephrase that part of the question in a form suitable for 11th/12th grade.

1) Show that if a number $b$ is expressed in the form $2^j - 1$ and $j$ is composite then $b$ is composite. (I.e. write out the general form for factoring this number.)

2) Show that if a number $b$ is expressed in the form $2^j + 1$ and $j$ is not a power of 2, then $b$ is composite. (I.e. write out the general form for factoring this number.)