Recall the problem: A five digit number has the strange property that putting a 1 after it gives you a number 3 times as large as putting a 1 in front of it. What is the number?

Find a) a fifth grade solution b) an Algebra I solution.

What could you change to give a new problem?

We want to find ways of extending this problem.

1. What happens if you change the 3 to some other number less than 10?
2. Look up or work out the algorithm for changing a repeating decimal to a fraction.
3. The following byway will inform your solution to question 4. Find integers $a$ and $b$ such that $\frac{a}{b} =$:

   1. $0.\overline{5}$
   2. $0.\overline{4}$
   3. $0.\overline{45}$
   4. $0.\overline{142857}$

5. Use calculators and also paper.

   (a) What is $7 \times 0.\overline{142857}$?
   (b) What is $1 \div 0.\overline{7}$?

4. Now let’s try to find for what lengths of $x$ we can find solutions to: A $k$-digit number has the strange property that putting a 1 after it gives you a number 3 times as large as putting a 1 in front of it. What is the number?

**Rubric**

The two solutions a) and b) were one point each. Problem 1) (find extensions) 2 points. The routing 2,3 were 1 point each. The reason I gave the ‘practice’ problems in 3 of converting decimals to rational numbers is that they provide a method of finding extensions. Only one student took this hint. But
I guess I didn’t phrase the problem clearly enough. Part 4 was worth up to 3 points and there were three significantly different levels of performance: checking values for low k; conjecturing what larger values of k work, giving some reasonable support for the conjecture.

**makeup** First let me be specific. We are only interested in integer solutions of these problems. I want you to just play some more with this problem. We will discuss the possible choices of k in class. Make up papers must include.

1. There are more solutions to the algebraic equation:

   \[ a(100000 + x) = 10x + 1 \]

   than to the original problem about putting 1’s in front and at the back. Why?

2. Find at least one more problem of form: ‘putting a in back of the number is the same as multiplying b times the result of putting a in front of the number’. Try to connect your example with the decimal expansions of fractions.