Rationals, Place Values, extensions

John T. Baldwin

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Recall the problem: A five digit number has the strange property that putting a 1 after it gives you a number 3 times as large as putting a 1 in front of it. What is the number?

Find a) a fifth grade solution b) an Algebra I solution.

What could you change to give a new problem?

We want to find ways of extending this problem.

1.What happens if you change the 3 to some other number less than 10?

2. Look up or work out the algorithm for changing a repeating decimal to a fraction.

3. The following byway will inform your solution to question 4. Find integers a and b such that $a/b =$:

1. $.\overline{9}$
2. $.\overline{4}$
3. $.\overline{45}$
4. $.\overline{142857}$
5. Use calculators and also paper.
   (a) What is 7×142857?
   (b) What is $1 \div 7$?

4. Now let's try to find for what lengths of x we can find solutions to: A k-digit number has the strange property that putting a 1 after it gives you a number 3 times as large as putting a 1 in front of it. What is the number?

Rubric

The two solutions a) and b) were one point each. Problem 1) (find extensions) 2 points. The routing 2,3 were 1 point each. The reason I gave the 'practice' problems in 3 of converting decimals to rational numbers is that they provide a method of finding extensions. Only one student took this hint. But

I guess I didn't phrase the problem clearly enough. Part 4 was worth up to 3 points and their were three significantly different levels of performance: checking values for low k; conjecturing what larger values of k work, giving some reasonable support for the conjecture.

makeup First let me be specific. We are only interested in integer solutions of these problems. I want you to just play some more with this problem. We will discuss the possible choices of k in class. Make up papers must include.

1. There are more solutions to the algebraic equation:

$$a(100000 + x) = 10x + 1$$

   than to the original problem about putting 1's in front and at the back. Why?

2. Find at least one more problem of form: 'putting a in back of the number is the same as multiplying b times the result of putting a in front of the number'. Try to connect your example with the decimal expansions of fractions.