More on Strange Numbers

MTHT400: due Oct. 4

This assignment has four parts.

1. Recall the following discussion from September 27. Suppose I want to find other lengths than 5 – say k so that one can solve the problem. There is a k digit number such that if I put a 1 after the number I get a number 3 times as large as the one obtained by putting a 1 in front of the number.

Thus, we are trying to find all $k$ such that:

$$10x + 1 = 3(10^k + x)$$

has a solution x which is a k-digit integer.

This means

$$10x + 1 = 3 \cdot 10^k + 3x$$

which implies

$$7x = 3 \cdot 10^k - 1.$$

That is 7 divides evenly into $3 \cdot 10^k - 1$. But since $10 \equiv 3 \bmod 7$, this is the same as

$$3 \cdot 3^k - 1$$

is divisible by 7. (You might prefer to write this as:

$$3 \cdot 3^k \equiv 1 \bmod 7.$$

That is, $3^{k+1} \equiv 1 \bmod 7$.)

What are all the possible choices of $k$ to satisfy this equation and thus answer the original problem.

2. Playing around with $\frac{2}{13} = .\overline{1538456}$, I found a solution to the puzzle: Find a 5 digit number such that if put a 1 after the number and multiply by 2 I get a number 7 times as large as the one obtained by putting a 1 in front of the number.

Like problem 1, show that if I replace 5 by k, I get that the possible $k$ are all numbers of the form $5 + 6r$ for any positive integer $r$.

3. Write a short note discussing a) what mathematical ideas are illuminated by different solutions to the strange number problem and b) the idea that mathematics can be motivated either externally (finding out how long a trip takes) or internally (solving questions that arise within mathematics). Should both of these motivations play a role in high school teaching?

4. The last part is completely open-ended. Try to work with various repeating decimals to find further riddles of this sort. If you find a method for transforming fractions m/n to problems this is great but I haven't gotten that far. This is just supposed to be 'fun'. See what you can do.